PROBLEM LIST

Question: The spaces $K_{p,q}(\mathbf{P}^n, \mathcal{O}(b), \mathcal{O}(d))$ have a Schur module decomposition. Set $H_{p,q}(\mathbf{P}^n, \mathcal{O}(b), \mathcal{O}(d))$ to be the submodule formed by the direct sum of all Schur submodules that are forced to arise via GL-equivariant Hilbert function considerations. At $d = 5$ it is known that $H_{14,1} \neq K_{14,1}$ (and similarly for $K_{13,2}$). Set $J_{p,q} = K_{p,q}/H_{p,q}$ the quotient. In the case of $K_{14,1}$, this is the Schur module

$$J_{14,1} \cong \mathbb{S}_{(30,25,20)} \oplus \mathbb{S}_{(30,24,21)}$$

As $d \to \infty$ is $J_{p,q}$ or $H_{p,q}$ the dominant term (or does it depend)? In other words, is $\dim H_{p,q} \gg \dim J_{p,q}$?

Question: For a non-degenerate prime ideal I in a polynomial ring S. Set $d = \deg V(I)$. Is there any known bound on $\operatorname{reg} I$ in terms of just d. The answer is yes, due to arguments based on Hochster-Ananayan, but the bound is non-constructive. What is an explicit bound?

Question: For a variety $X \subset \mathbf{P}^r$ with ideal sheaf $\mathcal{I}_X$, assuming $\mathcal{I}_X(d)$ is globally generated, is there a Bezout type bound $\operatorname{reg} \mathcal{I}_X \leq O(d^r)$ (or $O(d^{2r})$ etc)?

Question: Let X be a smooth variety embedded by the a line bundle $L_d = dA + P$ with A ample. Set $r_d = h^0(L_d) - 1$. For $d \gg 0$, we have positive constants C_1, C_2 such that $K_{p,q} \neq 0$ for $C_1 d^{q-1} \leq p \leq r_d - C_2 d^{n-1}$. Are there effective versions of C_1, C_2 and d for any class of varieties outside the Veronese or curve cases? Can one then describe C_1 and C_2 in terms of 'geometric data'?

Question: For a k-gonal curve C and line bundle L the Petri-map $\varphi\colon H^0(L) \otimes H^0(K_C \otimes L^{-1}) \to H^0(K_C)$ is known to be injective for generic curves. When it fails to be injective, $\dim \ker \varphi \geq 2(g - k + 1) - g$. Is this sharp for general curves with a g^k_1?

Question: The embedding $\mathbf{P}^2 \to \mathbf{P}^r$ by $\mathcal{O}(2)$ is well understood. For a smooth projective toric surface X, fix a minimal ample divisor A, by which we mean a divisor A for which any $D < A$ fails to be ample. For example, $\mathcal{O}(1,1)$ on $\mathbf{P}^a \times \mathbf{P}^b$. What can be said about the Betti table of $2A$?

Question: The asymptotic syzygy proof for Veronese uses a certain monomial construction to explicitly construct non-vanishing Koszul classes for $K_{p,q}$, see [EEL]. We call these *monomial syzygies*. Can this method be used for vanishing? Alternately, are similar computations possible in other examples (e.g., toric or monomial ideals)?

Question: What percentage of the space $K_{p,q}(\mathbf{P}^n, \mathcal{O}(d))$ are coming from the monomial syzygies? We expect the answer to be 0%, but what about if we consider the GL-orbit of the monomial syzygies?

Question: Can one explicitly lift the monomial syzygies form the Artinian case to the original case? This may be interesting already in the case of $\mathbf{P}^1$, for the $K_{p,1}$ syzygies of $\mathbf{P}^2$.

Question: Give any class of examples with more than linear vanishing for $K_{p,q}$ which does not come form known computations/duality arguments.

Question: Can one use Čech cohomology computations to explicitly compute $K_{p,1}$ on $\mathbf{P}^2$ using the description $K_{p,1}$ as the cohomology of exterior powers of the kernel bundle?

References

[EEL] L. Ein, D. Erman, R. Lazarsfeld, Asymptotics of random Betti tables, J. Reine Angew. Math. 702 (2015), 55-75.