Abstracts

Speaker: Bryden Cais
Title: Breuil–Kisin modules via crystalline cohomology

Abstract: Let k be a perfect field of characteristic p, and K a totally ramified extension of $W(k)[1/p]$. The theory of Breuil-Kisin modules provides a classification of stable lattices in crystalline p-adic representations of the absolute Galois group, G_K, of K via finite-height Frobenius modules over the power series ring $W(k)[[u]]$. On the other hand, the i-th integral p-adic étale cohomology of a smooth and proper formal scheme X over the ring of integers $\mathcal{O}_K$ in K provides a stable lattice in a crystalline p-adic G_K-representation, and so has a Breuil-Kisin module attached to it. In this case, it is natural to ask if the associated Breuil-Kisin module can be described in terms of the crystalline cohomology of X. Recent work of Bhatt, Morrow, and Scholze provides such a description after extending scalars to the period ring A_{inf}. In this talk, I will explain how to descend this result to obtain the Breuil-Kisin module over $W(k)[[u]]$ when $i < p-1$ and the crystalline cohomology of the special fiber of X is p-torsion-free in degrees i and $i+1$. This is joint work with Tong Liu.

Speaker: Richard Crew
Title: Isocrystals on a noetherian base

Abstract: Berthelot's theory of overconvergent isocrystals provides an analogue, for smooth schemes over a field of positive characteristic, of the notion of a local system. As in the classical case this category is equivalent to the category of modules over a suitable ring of differential operators. In this talk I will construct a generalization of Berthelot's rings of arithmetic differential operators, and use it to construct a category of p-adic local systems on any scheme of finite type over a field of positive characteristic.

Speaker: Veronika Ertl
Title: Comparison of crystalline syntomic and rigid syntomic cohomology for strictly semistable schemes.

Abstract: We prove a comparison isomorphism between Nekovár and Nizioł's syntomic cohomology and log rigid syntomic cohomology for a strictly semistable scheme with a nice compactification. Key points are a generalization of Große-Klönne's log rigid cohomology and the compatibility of crystalline and rigid Hyodo-Kato maps on the Frobenius eigenspaces. This is joint work with Kazuki Yamada.

Speaker: David Hansen
Title: Zariski-constructible sheaves on rigid spaces

Abstract: Constructible sheaves play an important role in the classical theory of étale cohomology. In this talk, I'll define an analogous class of so-called "Zariski-constructible" étale sheaves on rigid analytic spaces, and explain some of their basic properties. Along the way, I'll discuss some new comparison theorems.

Speaker: Kiran Kedlaya
Title: Old and new themes in p-adic cohomology

Abstract: One of the crowning achievements of mathematics in the 20th century is the resolution of Weil's conjectures on the number of rational points on algebraic varieties over finite fields. These conjectures have consequences through algebraic number theory, as well as in the theory of automorphic forms (the Ramanujan-Petersson conjecture), cryptography (computation of orders of class groups of curves over finite fields, which figure into modern public-key encryption and signature schemes), coding theory (error-correcting codes based on algebraic geometry over finite fields), etc. In addition, the methods developed for the proof themselves form the backbone of modern algebraic geometry, with further applications too numerous to describe here.

Often unnoticed in this story is the fact that the original approach to the Weil conjectures, via Grothendieck's idea of algebraizing the topological theory of covering spaces (leading to the concept of *étale cohomology*), is not the only possible approach. An alternate possibility is to use methods originating from Dwork's proof of one of the Weil conjectures (rationality of the zeta function); these were subsequently systematized into a theory of *p-adic cohomology* through the initial efforts of Berthelot and subsequent work of numerous authors. This construction provides complementary information to étale cohomology in many theoretical contexts; in addition, it provides important computational techniques unavailable from étale cohomology, some of which are widely deployed in modern computer algebra systems like `Magma` and `Sage`.

The backbone of p-adic cohomology is a form of de Rham cohomology for rigid analytic spaces; one may then imagine that by analogy with the de Rham comparison theorem relating singular and de Rham cohomology for C^∞ manifolds (or the Dolbeaut version for complex manifolds), one may be able to compare étale and de Rham cohomology for varieties over p-adic fields. This is true and forms the heart of *p-adic Hodge theory*. While many techniques are used in this subject, we will focus on the key concept of a *perfectoid space*, which plays a central role in recent work on this topic by Scholze, Kedlaya-Liu, Bhatt-Scholze-Morrow, et al. A perfectoid space is a certain type of nonarchimedean analytic space (technically an *adic space* in the sense of Huber) which transforms canonically between characteristic 0 and characteristic p; this provides a powerful mechanism for systematically using tools from positive-characteristic geometry (e.g., the Frobenius map) to make statements about geometry over p-adic fields.

The lecture series will consist of five standalone but related lectures, on the following topics.

1. **Analytic and algebraic de Rham cohomology:**
   The cohomology of differential forms was originally introduced in the concept of real differentiable manifolds (de Rham), and later transferred to complex analytic varieties (Dolbeaut) and algebraic varieties (Atiyah-Hodge, Grothendieck). We illustrate this progression; in the process, we see how additional structures that appear originally at the analytic level (e.g., the Hodge filtration, the Gauss-Manin connection in a family) acquire algebraic interpretations (e.g., through the work of Katz-Oda).
2. **Crystalline and rigid cohomology:**

The properties of zeta functions over finite fields of characteristic p are usually studied using étale cohomology with coefficients in the field $\mathbb{Q}_\ell$ for some prime $\ell \neq p$. However, this construction breaks down when $\ell = p$, and ideas from the theory of algebraic de Rham cohomology are needed to replace it. These ideas, developed initially by Berthelot, turn out to be closely related to the approach to zeta functions via p-adic analysis developed by Dwork.

3. **Computational applications:**

   Since 2000, a variety of methods have been developed that apply the ideas and methods of p-adic cohomology to problems of computational number theory. These include the determination of zeta functions for varieties over finite fields and (via methods of Chabauty, Coleman, and Kim) rational points for varieties over number fields.

4. **Perfectoid spaces:**

   Perfectoid spaces provide a consolidation of several key ideas in p-adic Hodge theory, such as the Fontaine-Wintenberger field of norms construction and the Faltings almost purity theorem. Moreover, several natural infinite towers of spaces admit inverse limits in the category of perfectoid spaces, giving rise to exotic but important new geometric structures.

5. **Applications of perfectoid spaces in p-adic Hodge theory:**

   We describe in detail how perfectoid spaces can be used to clarify and extend several key results in p-adic Hodge theory, such as the explicit description of étale fundamental groups of analytic spaces, and the comparison isomorphism between étale and de Rham cohomology for smooth proper spaces (without and with coefficients).

Speaker: Brian Lawrence
Title: Diophantine problems and a p-adic period Map

Abstract: We discuss a proof of the S-unit theorem and Faltings' theorem (finiteness of rational points on higher-genus curves) using p-adic Hodge theory and a p-adic period map.

Speaker: Ruochuan Liu
Title: Towards a p-adic Riemann-Hilbert correspondence

Abstract: I'll report the recent progress on the p-adic version of Riemann-Hilbert correspondence. Joint work with Xinwen Zhu.

Speaker: Tong Liu
Title: Fontaine-Messing theory over power series ring

Abstract: Let k be a perfect field of characteristic $p > 2$, $R = W(k)[\![t_1, \dots, t_d]\!]$ a power series ring and X a proper smooth scheme over R. I will discuss an ongoing project with Yong Suk Moon and Deepam Patel. Our goal is to extend Fontaine-Messing's theory form $W(k)$ to R, that is, to establish comparison between torsion crystalline cohomology and torsion étale cohomology of X.

Speaker: Yong Suk Moon
Title: Relative crystalline representations and p-divisible groups

Abstract: Let k be a perfect field of characteristic $p > 2$, and let K be a finite totally ramified extension over $W(k)[\frac{1}{p}]$. Kisin showed that any crystalline $\mathrm{Gal}(\overline{K}/K)$-representation whose Hodge-Tate weights lie in $[0, 1]$ arises from a p-divisible group over $\mathcal{O}_K$. We study the analogous problem for relative crystalline representations over the unramified base ring $R = W(k)[[t_1, ..., t_d]]$, and identify the relative crystalline representations which arise from a p-divisible group over R.

Speaker: Ananth Shankar
Title: The p-curvature conjecture and families of varieties.

Abstract: The Grothendieck-Katz p-curvature conjecture is an analogue of the Hasse principle for differential equations. It states that a set of arithmetic differential equations on a variety has finite monodromy if its p-curvature vanishes modulo p, for almost all primes p. We will discuss the case of families of varieties, and among other things, will prove that every simple closed loop has finite monodromy if the variety is a generic family of curves.

Speaker: Alberto Vezzani
Title: Cohomological realizations of rigid analytic motives

Abstract: We present the homotopical framework of motives applied to rigid analytic varieties and adic spaces. We show how it can be used to generalize some properties of classical cohomology theories and, when intertwined with the tilting equivalence, even to define new interesting ones.

Speaker: Liang Xiao
Title: Slopes of modular forms and ghost conjecture of Bergdall and Pollack

Abstract: For a p-adic modular form that is an eigenvalue of the U_p-operator, its slope is the p-adic valuations of the U_p eigenvalue. The slopes of modular forms were studied by Coleman, Gouvêa, Mazur, Buzzard, Calegari, and many mathematicians. Recently, Bergdall and Pollack formulated a conjecture that gives the precise slopes of all p-adic modular forms under the so-called Buzzard-regular condition. We give a reformulation in purely representation theory of $\mathrm{GL}_2(\mathbf{Q}_p)$, and explain its relation to p-adic local Langlands program.

Speaker: David Zureick-Brown
Title: p-adic cohomology and le Stum's Overconvergent Site

Abstract: Classically, the construction of rigid cohomology is a bit complicated and requires many choices. A recent advance is the construction by le Stum of an 'Overconvergent site' which computes the rigid cohomology of a variety X. This site involves no choices and so it is trivially well defined, and many things (like functoriality) become transparent. I'll explain this construction and several applications.