

# Spring Lecture Series N.27

## Harmonic Analysis, Multilinear Operations, and Schrodinger Operations (2002)

Principal Lecturer: Michael Christ, UC Berkeley

[A list of open problems: "Harmonic Analysis, Multilinear Operators, and Schrodinger Operators" \(PDF file\)](#)

### Invited Speakers:

Steven Hofmann (University of Missouri, Columbia)

*Title: TBA*

Nets Katz (Washington U. - St. Louis)

*Title: TBA*

Alexander Kiselev (University of Chicago)

*Title: Imbedded singular continuous spectrum for Schrödinger operators"*

**Abstract:** We construct examples of potentials decaying arbitrarily slower than at the Coulomb rate which lead to imbedded singular continuous spectrum of the corresponding Schrödinger operators.

Michael Lacey (Georgia Tech)

*Title: Quadratic Carleson Theorem*

**Abstract:** Consider the maximal function  $C_d f(x) := \sup_{\deg(p)=d} \left| \int f(x-y)e^{ip(y)} dy \right|$ , where the supremum is formed over all polynomials of a fixed degree. Motivations for considering this come from the case of  $d=1$ , in which case this is Carleson's maximal operator. In the higher degree case, a variety of results due to Ricci, Stein and Wainger have shown that the norms of oscillatory singular integrals can be taken to depend only on the degree of the polynomial. The conjecture that the maximal operator  $C_d$  is bounded, due to Stein, seeks a unification of these results with Carleson's theorem. I will discuss a proof of the boundedness of the operator  $C_2$ , which builds upon joint work with C. Thiele on Carleson's theorem and an theorem of E.M. Stein, and briefly describe Stein and Wainger's argument that for arbitrary degree,  $C_d$  is bounded if, and this is most essential, the linear term is taken to vanish in the polynomial  $p$ .

Michael Lacey (will also present a talk aimed to undergraduates on Wednesday, April 10)

*Title: Cryptography, Card Tricks, and Kangaroos*

**Abstract:** Cryptography is the basis of all forms of secure encoded communication. A commonly used cryptographic protocol is due to Diffie and Helman. I will explain this protocol in the lecture, and explain how this is related to a discrete form of the exponential function. This protocols can be broken if you can solve the "discrete logarithm problem." Of course the Diffie Hellman protocol is "secure" only to the degree that the discrete logarithm problem is "hard." A powerful method for solving the problem is the Kangaroo method of a famous cryptologist D.M. Pollard. And a fun card trick very nicely illustrates Pollard's Kangaroo method. Both will be explained in the lecture.

Fedor Nazarov (Michigan State University)

*Title: A few questions that we must answer before dealing with Navier-Stokes*

Wilhelm Schlag (CalTech)

*Title: TBA*

Andreas Seeger (Wisconsin)

*Title: Singular maximal functions in classes near  $L^1$*

Christoph Thiele (UCLA)

*Title: Nonlinear Fourier Analysis*

**Abstract:** Scattering and Inverse Scattering maps are widely regarded as nonlinear variants of the Fourier transform. For harmonic analysts this suggests to prove nonlinear variants of the basic estimates in Fourier analysis. Ultimately this leads to a better understanding of scattering theory. We are mainly interested in a nonlinear variant of Carleson's theorem. As of yet, this is unknown, but with Muscalu and Tao we have partial results such as a theorem in the Walsh case. In the talk we will give a brief introduction into the concept of nonlinear Fourier transform and discuss our results.

Stephen Wainger (University of Wisconsin),

*Title: Maximal functions on the discrete Heisenberg group with applications to ergodic theory*

**Contributions by:**

Arpad Benyi (University of Kansas)

*Title: Almost orthogonality and a class of bounded bilinear pseudodifferential operators*

Laura De Carli (University of Missouri, Columbia)

*Title: On the restriction properties of the fourier transform and related problems*

Marianne Korten (Kansas State)

*Title: Some results on the two-phase Stefan problem*

Norberto Laghi (University of Wisconsin)

*Title:  $L^2$  boundedness of restricted X-ray transforms in  $R^3$*

Jose Maria Martell (University of Missouri, Columbia),

*Title: Some extrapolation results for  $A_{\infty}$ -weights*

Irina Mitrea (Cornell),

*Title: On the global hölder regularity of conformal maps*

Kasso Okoudjou (Georgia Tech),

*Title: Embeddings of Some Classical Banach Spaces Into Modulation Spaces*

Malabika Pramanik (University of Wisconsin)

*Title: Decay of weighted oscillatory integrals in  $R^2$*

Igor E. Pritsker (Oklahoma State),

*Title: A discrete Pompeiu problem*

Abdumalik Rakhimov (University of Uzbekistan)

*Title: On the localization of the multiple trigonometric Fourier series of distributions*

Jorge Rivera-Noriega (University of Illinois)

*Title: Parabolic measure in non-cylindrical domains*

Svetlana Roudenko (Michigan State University)

*Title: Matrix Weights on Function Spaces*

Dmitry Ryabogin (University of Missouri, Columbia)

*Title: Some progress on the logarithmic problem for singular integrals with rough kernels*

Alexander Stokolos (University of Connecticut)

*Title: Maximal Function With Respect to Convex Set*

Nasser Towghi (University of Connecticut)

Chan Woo Yang (Univ. of Wisconsin),

*Title:  $L^p$  improving estimates for some classes of Radon transforms*

Shijun Zheng (University of Maryland)

*Title: Spectral Multiplier Theorems for 1-D Schrödinger operators*