

Spring Lecture Series N.28

The Andrews-Curtis and the Poincare Conjectures (2003)

Principal Lecturer: Andrew Casson (Yale University)

Invited Speakers:

Stephen Bigelow

Title: Representations of the symmetric group

Abstract: A classical problem in representation theory is to classify the representations of the symmetric group. There is a beautiful solution over a field of characteristic zero. The behavior in characteristic p is related to the behavior of the Hecke algebra when the parameter q is a primitive p th root of unity. I will discuss a topological approach to understanding this problem and its relation to the classical problem.

Martin Bridson

Title: Curvature, complexity and subgroups

Abstract: We show that many natural classes of groups of homeomorphisms of the plane are (abstractly) isomorphic to groups of homeomorphisms of the circle. There are connections with the theory of flows on 3-manifolds, generalized braid groups, and Zimmer's conjecture.

Danny Calegari

Title: Circular groups and planar groups

Abstract: In this talk I'll discuss recent results about the nature of the groups that cluster around the concept of non-positive curvature in group theory. Most of these results concern issues of complexity and subgroup structure. Much of the discussion will focus on free-by-cyclic groups, whose subtle diversity is both illuminated and illuminating in this context. If time allows, I shall also discuss the complexity of certain families of Andrews-Curtis trivialisable balanced presentations.

Nathan Dunfield

Title: Laminations of 3-manifolds and groups of homeomorphisms of the circle

Abstract: If M is an atoroidal 3-manifold with a taut foliation, Thurston showed that $\pi_1(M)$ acts faithfully on a circle. In this talk, I will discuss why certain other classes of essential laminations also give rise to circle actions. (Joint work with D. Calegari).

Cameron Gordon

Title: Surface subgroups of Coxeter groups

Abstract: We characterize those Coxeter groups that contain a surface subgroup. It follows from this characterization that a Coxeter group either contains a surface group or is virtually free. In particular, Gromov's question as to whether a 1-ended word hyperbolic group contains a hyperbolic surface group has an affirmative answer in the class of Coxeter groups. (Joint work with Darren Long and Alan Reid.)

Alan Reid

Title: Arithmetic rational homology spheres

Abstract: We will discuss how common it is that an arithmetic hyperbolic manifold or orbifold is a rational homology sphere.

Martin Scharlemann

Title: There are no unexpected tunnel number one knots of genus one

Abstract: We show that the only knots that are tunnel number one and genus one are those that are already known: 2-bridge knots obtained by plumbing together two unknotted annuli and the satellite examples classified by Eudave-Muñoz and by Morimoto-Sakuma. The principal new tools are a useful way of defining width for a 3-valent graph in S^3 , and a controlled way of loading the knot onto a neighborhood of such a graph. We analyze how the knot loading allows the graph to be thinned and show that eventually either the graph contains an unknot or the knot tunnel can be pushed onto the Seifert surface. In either of these circumstances the result (known as the Goda-Teragaito Conjecture) was already known.

Zlil Sela

Title: Low dimensional topology and aspects of the first order theory of a free group

Abstract: We study sets of solutions to equations over a free group, projections of such sets, and the structure of elementary sets defined over a free group. The structure theory we obtain enable us to answer some questions of A. Tarski's, and classify those finitely generated groups that are elementary equivalent to a free group. Connections with low dimensional topology, a generalization to (Gromov) hyperbolic groups, and further aspects of the first order theory of free (and hyperbolic) groups will also be discussed.

Peter Shalen

Title: Smallish knots

Abstract: Let Σ be a non-Haken irreducible 3-manifold. A knot K in Σ is said to be smallish if its exterior is irreducible and contains no bounded essential surface whose boundary components are meridian curves. I will describe recent progress in an on-going joint project with Culler, Dunfield and Jaco directed at proving the conjecture that every non-Haken irreducible Σ contains a smallish knot. The conjecture, in addition to its intrinsic interest, forms part of a program for proving the Poincare Conjecture.

Contributions by:

Speaker: Ian Agol

Title: Marden's conjecture and exceptional Dehn fillings

Abstract: Marden's conjecture states that a complete hyperbolic 3-manifold with finitely generated fundamental group is tame, i.e. the interior of a compact 3-manifold. Given a manifold with torus boundary and hyperbolic interior (a hyperbolic knot complement), an exceptional Dehn filling is one which is reducible, or has finite fundamental group, or non-word-hyperbolic fundamental group. Assuming Marden's conjecture, we show that there are only finitely many 1-cusped hyperbolic knot complements which have > 8 exceptional Dehn fillings. It is conjectured that there are only finitely many with > 6 exceptional fillings, and an explicit list of such manifolds is conjectured to be the only examples. We will also show that for a non-compact hyperbolic manifold M with $b_1(M) > 2$, the volume of M is $\geq 2\pi \cdot v_3 / \sqrt{3}$, where v_3 is the volume of a regular ideal tetrahedron in H^3 . The common thread of these two theorems is a result of Anderson, Canary, Culler and Shalen which gives an improved Margulis lemma for tame free groups.

Speaker: Mark Brittenham

Title: Knots with unique minimal genus Seifert surface

Abstract: We show how to build families of knots with unique minimal genus Seifert surfaces, and apply this to construct hyperbolic knots with depth greater than one.

Speaker: Sergio Fenley

Title: 3-manifolds, laminations and group actions

Abstract: Manifolds with essential laminations have fantastic properties, for example they are irreducible and have universal cover homeomorphic to \mathbb{R}^3 . Suppose that a 3-manifold M has an essential lamination L . If there are leaves isolated on both sides, blow each one to an I-bundle of leaves. Then in the universal cover of M , the leaf space of the lifted lamination is an example of a non Hausdorff tree and the fundamental group acts on it. After a further modification one produces a non-trivial action on an actual tree. Some specific group actions on trees coming from manifolds obtained by Dehn surgery on torus bundles over the circle can be analyzed in detail. These give information about the existence problem for essential laminations in 3-manifolds.

Speaker: Hiroshi Goda

Title: Heegaard splitting for sutured manifolds and Circle valued Morse theory for knots and links

Abstract: We show Circle valued Morse theory for knots and links in the (homology) 3-sphere. A handle decomposition corresponding to a circle valued Morse function can be regarded as Heegaard splitting for sutured manifold. We discuss behaviors of the Heegaard genera and its estimate using knot invariants.

Speaker: Shelly Harvey

Title: Some remarks on the Virtual Betti Number of a 3-manifold

Abstract: It is conjectured that every closed 3-manifold M has finite covers with arbitrarily large first betti number. We discuss this question for arbitrary abelian covers. In particular, we give a sequence of algebraic and topological properties which guarantee the existence of a cyclic cover with "large" first betti number and discuss some particular examples.

Speaker: Ben Klaff

Title: Boundary slopes of knots in closed 3-manifolds with cyclic fundamental group

Abstract: We show that if N is a closed 3-manifold with odd cyclic fundamental group and K is a (tame) knot in N such that the exterior of K is irreducible, then at least one of the following holds: (1) for every framing of the knot K , there exists a boundary slope whose absolute value is greater than one; or (2) K is an iterated cable of a knot whose exterior is a solid torus. The main argument in the proof, based on N. Dunfield's argument in the case where K is hyperbolic, brings to bear some deep results in the theory of character varieties of hyperbolic 3-manifolds.

Speaker: Jason Manning

Title: Quasi-actions on trees and the geometry of pseudocharacters

Abstract:

We show how a nontrivial pseudocharacter on a group gives rise to a quasi-action on a tree. This gives rise to examples of exotic quasi-actions on bushy (infinite valence) trees. We also give some examples of groups which cannot quasiact coboundedly on any infinite tree.

<http://front.math.ucdavis.edu/math.GR/0303380>

Speaker: Roman Mikhailov

Title: Augmentation powers, group homologies and localizations

Abstract: We consider generalized polinomial filtrations in group cohomologies and Dwyer's filtrations (which play important role in the 4-dimensional topology), consider transfinite conditions on this filtrations. Also we define transfinite parafree groups and study some "homologically wild" groups. (This is a joint work with I.B.S.Passi)

Speaker: Jennifer Schultens

Title: Some topology and algebra of graph manifolds

Abstract: Graph manifolds possess enough structure to allow for many types of calculations. Under certain orientability assumptions, enough is known about the structure of the Heegaard splittings of a graph manifold to determine its Heegaard genus. Joint work with Richard Weidmann shows that the difference between the Heegaard genus and the rank of the fundamental group of a graph manifold can be arbitrarily large.