Photoelectron waiting times and atomic state reduction in resonance fluorescence

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Photoelectron counting sequences for single-atom resonance fluorescence are studied. The distribution of waiting times between photoelectric counts is calculated, and its dependence on driving-field intensity and detection efficiency is discussed. The photoelectron-counting distribution is derived from the waiting-time distribution. The relationship between photoelectron counting sequences and photon emission sequences is discussed and used to obtain an expression for the reduced state of the atom during the waiting times between photoelectric counts. The roles of irreversibility and the observer in atomic state reduction are delineated.

I. INTRODUCTION

The fluorescent photons emitted by a single coherently driven two-level atom exhibit the nonclassical property of photon antibunching.[1-3] The antibunching of fluorescent photons is seen in temporal correlations between photoelectric counts; the detection of one photon makes the detection of a second, after just a short delay, improbable. Photon antibunching is traditionally defined in terms of the degree of second-order temporal coherence $g^{(2)}(t,t+\tau)$. This is the joint probability for recording photoelectric counts in the intervals $[t,t+\Delta t)$ and $[t+\tau,t+\tau+\Delta t)$, normalized by the probability for two independent photoelectric counts. For antibunched light the joint probability for recording photoelectric counts closely spaced in time falls below the probability for statistically independent counts (separated by a time longer than the coherence time); thus, $g^{(2)}(t,t)<1$.

The antibunching of fluorescent photons is also reflected in the sub-Poissonian character of the probability density $p(n,t,t+T)$ for recording n photoelectric counts in the interval $[t,t+T)$.[4] $p(n,t,t+T)$ can be derived from $g^{(2)}(t,t+\tau)$, although the detailed algebraic relationship is quite complicated. Both $g^{(2)}(t,t+\tau)$ and $p(n,t,t+T)$ have been calculated for single-atom resonance fluorescence by a number of workers.[1-10] Because of the complexity of general expressions in the time domain, some workers only give the Laplace transform for the photoelectron counting distribution, or give explicit time-dependent expressions only for limiting cases, such as short and long counting times.

Recent theoretical work on "quantum jumps"[11,12] has drawn attention to the distribution of waiting times between photon emissions as another useful quantity for characterizing photon statistics—in terms of measured quantities, the distribution of waiting times between photoelectrons. By "waiting time" we mean the time τ between a photoelectric count recorded at time t, and the next, recorded at time $t+\tau$. If photoelectron sequences can be described by a Markov birth process, a single conditional probability density $w(\tau|t)$ specifies the distribution of waiting times between every pair of photoelectrons. We call this the photoelectron waiting-time distribution. Photoelectron waiting times for coherent light are exponentially distributed.[13] Antibunching implies that photons tend to be separated in time. The distribution of waiting times should then tend to peak around the average time between photoelectric counts.

Photoelectron waiting times are certainly not new to the field of photon statistics. Indeed, when a time-to-amplitude converter is used for a delayed coincidence measurement, the raw data provide the distribution of waiting times between photoelectric counts. However, when the count rate is sufficiently low, this distribution is proportional (aside from dead-time corrections) to $g^{(2)}(t,t+\tau)$.[14] This relationship provides the technique used to measure $g^{(2)}(t,t+\tau)$ in the experiments of Kimble *et al.*[3] on photon antibunching in resonance fluorescence. Thus, the waiting-time distribution and its relationship to $g^{(2)}(t,t+\tau)$ are known. But the waiting-time distribution has not been mentioned until recently[12,15,16] in the large theoretical literature on resonance fluorescence. This is a deficiency, since it provides a clearer physical picture of photon emission sequences, corresponding photoelectron counting sequences, and their nonclassical properties, than $g^{(2)}(t,t+\tau)$. In this paper we revisit the problem of single-atom resonance fluorescence and focus attention on the waiting-time distribution. (We will discuss waiting times between photon emissions as well as between photoelectrons. When the distinction is not important we simply refer to "the waiting times" or "the waiting-time distribution.")

There are probably two main reasons for the lack of attention paid to $w(\tau|t)$ in early work on resonance fluorescence. The first is that $g^{(2)}(t,t+\tau)$, not $w(\tau|t)$, is the quantity accessible to measurement. It might be asked, why not use a time-to-amplitude converter to measure the quantity it gives directly, the photoelectron waiting-time distribution $w(\tau|t)$? The problem is that photoelectric detection is very inefficient. The average time between photoelectric counts is unavoidably much longer than the correlation time of the fluorescent light. Then $w(\tau|t)$ is proportional to $g^{(2)}(t,t+\tau)$; $w(\tau|t)$ can be measured, but only when it effectively reduces to $g^{(2)}(t,t+\tau)$. Actually, the proportionality between these quantities does not hold for all τ, but it holds over many correlation

Subnatural linewidth averaging for coupled atomic and cavity-mode oscillators

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We calculate the spontaneous-emission spectrum and the spectrum of weakly driven fluorescence for a two-level atom coupled to a resonant-cavity mode. For strong atom-cavity coupling the spectra split into two peaks that can have subnatural linewidths. If the cavity linewidth is negligible, the spontaneous-emission spectrum has half the radiative linewidth of the atom; the spectrum of weakly driven fluorescence shows an additional 36% squeezing-induced narrowing. These effects can be observed using coupled-field and collective-polarization oscillators excited in a cavity containing N two-level atoms.

The spontaneous-emission rate for an atom in free space can change when the atom radiates inside an electromagnetic cavity. The emission rate is reduced if the cavity subtends a large solid angle at the atom and has no resonant modes into which the atom can emit.[1] It is increased when the atom couples strongly to a resonant mode of the cavity.[2] These effects are explained by perturbation theory; the altered emission rate is obtained from Fermi's golden rule using a density of states modified to account for the cavity boundary conditions. However, when the coupling between the atom and the cavity mode is so strong that a photon emitted into the cavity is likely to be reabsorbed before it escapes, perturbation theory cannot be used. Previous authors have studied atomic decay under these conditions assuming that spontaneous emission to modes other than the privileged cavity mode is negligible. Haroche and Raymond[3] show that an initially excited atom undergoes single-quantum Rabi oscillations which decay at a rate determined by the cavity Q. Sanchez-Mondragon *et al.*[4] derive a double-peaked "spontaneous-emission" spectrum by convolving the single-quantum Rabi oscillation in a lossless cavity against an exponential detector response function. This is not, however, a spontaneous-emission spectrum in the usual sense of irreversible decay into a reservoir of vacuum modes; in particular, linewidths are assigned by the detector response function and are not radiative in origin.

In this paper we consider the interaction between a two-level atom and a resonant cavity mode that subtends a *small* solid angle at the atom so that the spontaneous-emission rate γ into free-space is not negligible compared to the photon decay rate 2κ from the cavity. We derive the spontaneous emission spectrum and spectrum of weakly driven fluorescence measured by observing the light emitted (scattered) by the atom into free space. Our derivation places no restriction on the coupling strength g between the atom and the cavity mode. For $\kappa \gg g \gg \gamma/2$, we recover the increased linewidth associated with cavity-enhanced spontaneous emission. In the strong-coupling limit, $g \gg \kappa, \gamma/2$, the spontaneous-emission spectrum and the spectrum of weakly driven fluorescence are doublets, similar to those obtained by Sanchez-Mondragon *et al.*[4] However, our spectra have meaningful radiative linewidths. For $\kappa \ll \gamma/2$ they have linewidths equal to one-half and one-third the free-space radiative linewidth of the atom, respectively.

The master equation describing the resonant interaction between a two-level atom and a single cavity mode, including both atomic and cavity loss, is given by

$$\dot{\rho}=(1/i\hbar)[\hat{H},\rho]+(\gamma/2)(2\hat{\sigma}_-\rho\hat{\sigma}_+-\hat{\sigma}_+\hat{\sigma}_-\rho-\rho\hat{\sigma}_+\hat{\sigma}_-)$$
$$+\kappa(2\hat{a}\rho\hat{a}^\dagger-\hat{a}^\dagger\hat{a}\rho-\rho\hat{a}^\dagger\hat{a}) , \tag{1}$$

where $\hat{H}=i\hbar g(\hat{\sigma}_-\hat{a}^\dagger-\hat{\sigma}_+\hat{a})$ and ρ is the reduced density operator in the interaction picture; $\hat{a}^\dagger$ and $\hat{a}$ are creation and annihilation operators for the cavity mode, and $\hat{\sigma}_-$, $\hat{\sigma}_+$, and $\hat{\sigma}_z$ are Pauli pseudospin operators for the atom. Equation (1) describes a composite atom-cavity-mode system that radiates via two distinct channels: by the coupling of the atom to free-space modes (the term proportional to $\gamma/2$), and by loss through the cavity mirrors (the term proportional to κ). We will calculate spectra for the light emitted by the atom into free space.

To describe spontaneous emission for arbitrary values of g, κ, and $\gamma/2$, we solve Eq. (1) in the three-state basis $|+\rangle|0\rangle, |-\rangle|1\rangle, |-\rangle|0\rangle$, where $|+\rangle$ and $|-\rangle$ are the upper and lower states of the atom and $|1\rangle$ and $|0\rangle$ are the one-photon and zero-photon Fock states of the field. In this basis ρ has eight independent matrix elements. The equations of motion for these matrix elements can be written as two sets of coupled equations for operator expectation values,

$$\langle \dot{\hat{a}} \rangle = g\langle \hat{\sigma}_- \rangle - \kappa\langle \hat{a} \rangle , \tag{2a}$$

$$\langle \dot{\hat{\sigma}}_- \rangle = -g\langle \hat{a} \rangle - (\gamma/2)\langle \hat{\sigma}_- \rangle , \tag{2b}$$

and

Normal-Mode Splitting and Linewidth Averaging for Two-State Atoms in an Optical Cavity

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An investigation of the radiative processes for a collection of N two-state atoms strongly coupled to the field of a high-finesse optical cavity is presented. Observations of the spectral response of the composite system to weak external modulation reveal a coupling-induced normal-mode splitting. Linewidth averaging leads to linewidths below the free-space atomic width.

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The radiative processes of atoms in the presence of boundaries such as provided by a resonant cavity have been investigated in recent years within the context of cavity quantum electrodynamics.[1] While the free-space decay of an atom is characterized by the Einstein A-coefficient γ, the interaction of an atom with a cavity introduces three new rates not present for atoms in free space; these rates are the cavity damping rate κ, the decay rate γ' into continuum modes other than those of the cavity, and the coupling rate g, where g characterizes the oscillatory exchange of excitation between the atom and the field of the cavity. For weak coupling of the atom to the resonator ($\gamma' \ll g^2/\kappa \ll \kappa$), large enhancements[2] and suppressions[3] of atomic spontaneous emission have been observed[1] with cavities encompassing a large fraction f of the total 4π solid angle of free space ($\gamma' \ll \gamma$), in agreement with a perturbative description incorporating the cavity-modified density of states. On the other hand, for strong coupling ($g \gg \gamma', \kappa$) the nonperturbative nature of the interaction requires a description not from the viewpoint of the altered radiative processes of the atom or cavity alone, but rather in terms of the dynamics of the composite atom-field entity.[4,5] While impressive investigations of this underdamped regime have been conducted in the microwave domain with Rydberg atoms for which $g^2/\kappa \gg \kappa \gg \gamma'$,[6,7] complimentary studies in the optical domain have been largely absent since the coherent coupling rate g is usually dominated either by κ or by γ'. The interest in optical studies arises not only because of the possibility for direct field measurements, but also because of the opportunity for investigations in the strongly coupled regime with atomic dissipation entering as an important process.

Within this context we present in this Letter direct spectroscopic measurements of the normal-mode splitting for the oscillator system formed by the collective atomic polarization of N two-state atoms strongly coupled to a single mode of a high-finesse optical cavity.[8,9] Although $f \ll 1$ and hence $\gamma' \cong \gamma$, the experiments are nonetheless carried out in a regime for which $g\sqrt{N} \gg \gamma > \kappa$ (with $g\sqrt{N}$ the effective coupling rate in the limit of a weak intracavity field)[10,11] and hence in a regime for which a photon emitted into the cavity by the atomic polarization is likely to be absorbed and reemitted many times before it escapes. By recording the spectral response of the composite atom-cavity system to a weak external probe field, we observe a distinctive doublet symmetrically split about the otherwise common frequency of atoms and cavity and find that this normal-mode splitting is in quantitative agreement with the predicted eigenfrequencies over a range of intracavity atomic number $20 \leq N \leq 600$. Our observations at optical frequencies are thus of the vacuum-field Rabi splitting[12] extended from the one-atom case to the situation $N > 1$ and follow in the spirit of previous time-domain investigations of collective atom-cavity oscillations for Rydberg atoms in a microwave cavity.[13] For each of the two peaks of the split doublet, we also observe subnatural linewidths due to a dynamical linewidth averaging which results from the strong coherent coupling of the collective atomic polarization to the cavity mode.[5,14] Linewidth reductions of 25% relative to free-space atomic decay are recorded over a wide range of operating conditions. Finally, we explore the behavior of the normal-mode structure as the atomic and cavity resonances are detuned, and observe that the line positions and widths approach values characteristic of decoupled atoms and cavity.

Our starting point is an analysis of the eigenvalue structure for the composite system of N two-state atoms coupled to a single cavity mode. In the limit of weak intracavity field and for coincident cavity resonance frequency ω_C and atomic transition frequency ω_A, the eigenvalues obtained either from the master equation for a single atom[14] or from the Maxwell-Bloch equations for $N \gg 1$[10] are $\lambda_0 = -\gamma$ and

$$\lambda_{\pm} = -\tfrac{1}{4}(2\kappa + \gamma/\Gamma) \pm [\tfrac{1}{16}(2\kappa - \gamma/\Gamma)^2 - \kappa\gamma C/\Gamma]^{1/2}, \tag{1}$$

where λ_0 gives the decay of the atomic inversion and $\lambda_{\pm}$ describe the normal modes formed from the intracavity field and the collective atomic polarization. Equation (1) is written in a rotating frame at the common frequency $\omega_C = \omega_A$; $\Gamma \equiv \gamma/2\gamma_{\perp}$ with $\gamma_{\perp}$ the transverse decay rate; and $C = \Gamma N g^2/\kappa\gamma$ is the atomic cooperativity parameter of optical bistability, with $g = (\mu^2 \omega_C/2\hbar\epsilon_0 V)^{1/2}$,

Ringing revivals in the interaction of a two-level atom with squeezed light

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The Jaynes–Cummings interaction of a two-level atom with the radiation field is studied when the radiation is initially in a strongly squeezed coherent state. The dynamic response of the atomic inversion shows echoes after each revival when the squeezed coherent state exhibits an oscillatory photon-counting distribution due to the phase-space interference effect. The sensitivity of the dynamic behavior to approximations used in computing the atomic inversion is discussed. Comparison is made with the intensity-dependent interaction model of Buck and Sukumar [Phys. Lett. 81A, 132 (1981)]; this model does not exhibit echoes. The mean, variance, and entropy for the photon-number distribution are calculated and found to show behavior similar to that of the atomic inversion.

1. INTRODUCTION

The Jaynes–Cummings model of optical resonance[1] describing the interaction of a single two-level atom with a single mode of the radiation field has predicted a number of interesting features in the dynamical behavior of the atomic inversion.[2–4] Much attention has focused on the collapse and revival of Rabi oscillations because this effect provides evidence for the granularity of the radiation field.[3,4] Experimental realizations of the Jaynes–Cummings model have been obtained by using Rydberg atoms interacting with the radiation field in a high-Q microwave cavity.[5] Recently observations on the collapse and revival of Rabi oscillations were reported.[6] In this paper a new feature in the dynamical behavior of the atomic inversion is studied, with the radiation field prepared in a strongly squeezed coherent state whose photon-counting distribution is oscillatory.[7] Under these conditions, the collapse following each revival has an oscillatory envelope (echoes), a phenomenon that we call ringing revivals.

Milburn has studied the interaction of a two-level atom and a single mode of the radiation field with the field prepared in a squeezed coherent state.[8] He showed that the collapse time depends on the direction of the squeezing and found that for certain squeezed states the response of the atom is similar to that for chaotic radiation. However, Milburn restricted his study to states for which the coherent contribution to the photon-number variance is dominant. The new behavior described in this paper is obtained with squeezed coherent states for which the squeezed contribution to the photon-number variance is dominant.

Our results for the atomic inversion are based on the numerical evaluation of the series[1]

$$w(t) = -\frac{1}{2}\sum_{n=0}^{\infty} P(n)\cos(2\lambda t\sqrt{n}), \tag{1}$$

where $P(n)$ is the photon-number distribution for the initial state of the radiation field and λ is the coupling constant for the atom–field interaction. The atom is assumed to be in its ground state initially. An exact analytical evaluation of this sum is not possible even for an initial coherent state, although approximate expressions that reproduce the general character of the revivals have been obtained.[3,4] We compare the exact numerical evaluation of Eq. (1) with various approximations when $P(n)$ corresponds to a squeezed coherent state.

The plan of the paper is as follows: In Section 2 we briefly review the oscillatory nature of $P(n)$ for squeezed coherent states. We study the corresponding dynamical response of the atomic inversion in Section 3. The exact numerical evaluation of $w(t)$ is presented, demonstrating the occurence of ringing revivals, and the origin of the ringing behavior is discussed. In Section 4 we obtain a closed analytical expression for $w(t)$ in the harmonic approximation, in which the square root in the argument of the cosine in Eq. (1) is expanded to first order. The ringing of the revivals is lost in this approximation. Expansion of the square root to second order recovers the ringing behavior. The summation formula that yields an analytical result in the harmonic approximation may also be used to derive an exact integral representation for $w(t)$ when the radiation field is initially in a squeezed coherent state. This integral representation is given in Appendix A. We study the photon statistics and entropy for the field in Section 5. Our results are summarized in Section 6.

2. SQUEEZED COHERENT STATES

Squeezed coherent states are now quite familiar in quantum optics; we simply state their definition[9] and refer the reader to two recent collections of papers[10] for further details and a review of current activity regarding these states.

The squeezed coherent states for a single-mode radiation field can be obtained from the vacuum $|0\rangle$ as

$$|\alpha, \xi\rangle \equiv D(\alpha)S(\xi)|0\rangle, \tag{2}$$

where $S(\xi)$ is the squeezing operator

$$S(\xi) = \exp[\tfrac{1}{2}(\xi^* a^2 - \xi a^{\dagger 2})] \tag{3}$$

and $D(\alpha)$ is the displacement operator

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Nonclassical effects in optical spectra

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The incoherent part of the Mollow spectrum for resonance fluorescence has a subnatural linewidth in the weak-field limit. We show that this is due to squeezing of the fluctuations in the induced atomic dipole. The reduced linewidth persists for driving field intensities of approximately 12% of the saturation intensity, where Rabi sidebands begin to appear. We find a similar linewidth narrowing in the transmitted and fluorescent spectra for a single atom in a weakly driven resonant cavity. In this system single-quantum frequency splitting can produce a two-peaked spectrum. Both peaks have a narrowed linewidth. The spectrum of the transmitted light shows a second nonclassical effect that is due to squeezing. A spectral hole may appear at line center, giving a two-peaked spectrum even when there is no frequency splitting.

1. INTRODUCTION

Nonclassical electromagnetic fields have generated much interest in recent years. These fields show statistical properties that are incompatible with the language of classical statistical optics. They require a quantum-mechanical description. Two examples of the properties shown by nonclassical fields are photon antibunching and squeezing. Photon antibunching is defined by a nonclassical form for the degree of second-order coherence.[1,2] Squeezing is defined by reduced quantum fluctuations, below those for the vacuum state, in one quadrature phase amplitude of the field.[3] Both of these effects are understood theoretically and have been observed in several experiments. In this paper we discuss nonclassical effects that may be observed in the inelastic or incoherent part of the optical spectrum. Optical spectra may show reduced linewidths or spectral holes, which arise because one quadrature phase of the optical field is squeezed.

Recently Carmichael *et al.* calculated the resonance fluorescence spectrum for an atom interacting with a broadband squeezed vacuum.[4,5] They showed that this spectrum can have one Lorentzian component with a subnatural linewidth. We shall show that a subnatural linewidth also occurs in ordinary resonance fluorescence, in the absence of the squeezed vacuum. The linewidth narrowing is again due to squeezing, but the mechanism is different. In this case squeezed light does not irradiate the atom; it is produced in the interaction between the driven atom and the modes of the usual vacuum. The amount of linewidth narrowing is not so remarkable as in the previous work, but the effect may be easier to realize experimentally.

The mechanism for linewidth narrowing that we describe is not restricted to resonance fluorescence. It is reflected in a general relationship between the optical spectrum and the spectrum of squeezing. It follows that linewidth narrowing will be seen in many systems that show squeezing. We also present results for the interaction of a single two-level atom with a weakly driven optical cavity mode.[6,7] Linewidth narrowing is seen both in the light transmitted by the cavity and in the atomic fluorescence out the sides of the cavity. When the coupling between the atom and the cavity mode is sufficiently strong, the transmitted and fluorescent spectra may be double peaked. The two peaks arise from the splitting of the degenerate single-quantum states, an excited atom with no photons in the cavity mode, and an unexcited atom with one photon in the cavity mode. Double-peaked spectra result if this level splitting is large enough to overcome the effects of cavity and atomic damping. When these conditions are met we shall refer to the phenomenon of single-quantum frequency splitting.[8–12] (To us, the name vacuum Rabi splitting, used by previous authors, is inappropriate.) Both peaks of the spectra resulting from single-quantum frequency splitting show a reduced linewidth.

The cavity system illustrates a second nonclassical effect, not seen in resonance fluorescence. The spectrum of the transmitted light may have two peaks even though the conditions for single-quantum frequency splitting are not met. A hole may appear at line center because of squeezing of the fluctuations in one quadrature phase amplitude of the field. Lugiato found similar holes in transmitted spectra for optical bistability in the good-cavity limit and noted their significance as nonclassical effects.[13] We are now able to identify the origin of these spectral holes.

In Section 2 we discuss linewidth narrowing in resonance fluorescence and explain this narrowing in terms of a relationship between the optical spectrum and the spectrum of squeezing. In Section 3 we present spectra for a single atom in a driven optical cavity. These spectra show the same linewidth narrowing. Spectral holes induced by squeezing are discussed in Section 4.

2. SQUEEZING-INDUCED LINEWIDTH NARROWING

In resonance fluorescence a two-state atomic transition is excited on resonance by a strong coherent field. The expression for the spectrum of the fluorescent light has been calculated by Mollow[14] and by Cresser and others (see the review by Cresser *et al.*[2]). This spectrum has two components: a coherent, or elastically scattered, component, which corresponds to classical reradiation from the induced atomic dipole, and an incoherent component that is due to quantum fluctuations. For excitation intensities much

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Regular Papers

Single-Atom Optical Bistability

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Abstract—**Absorptive optical bistability is shown to exist for a single two-level atom inside a resonant optical cavity. Solutions for the quantum mechanical density operator are obtained numerically for a parameter regime at the interface between the quantum limit, in which quantum mechanical noise invalidates the semiclassical prediction of bistability, and the classical limit, in which quantum noise is a negligible perturbation on semiclassical results. Bimodal photon number distributions and Q functions are obtained, and two-state transition rates are calculated.**

THE original proposal for absorptive optical bistability was based on a semiclassical understanding of a saturable absorber. In this view, the absorber is described by an intensity-dependent absorption coefficient, and a simple argument leads to the prediction of bistable transmission for an interferometer containing a saturable medium [1]. This argument becomes inappropriate, however, when the absorber is reduced to a single atom. The saturation of a single two-level atom is accompanied by large quantum fluctuations; these are evidenced by the dominant *incoherent* scattering in resonance fluorescence at saturating intensities. The behavior of the electromagnetic field inside an interferometer containing a *single* atomic absorber must be described by a complete quantum mechanical theory.

In its simplest form, such a theory must treat a lossy driven mode oscillator interacting with a lossy (due to spontaneous emission out the sides of the cavity) atomic transition. The model is formulated as an extension of the Jaynes–Cummings model to include dissipation and an injected field [2]. If quantum fluctuations are neglected, the semiclassical model for absorptive optical bistability is recovered. However, does optical bistability exist when quantum fluctuations are included, and if it does, under what conditions does it exist? Recent work by Sarkar and Satchell suggests that absorptive optical bistability does not exist for a single atom [3]. McCall and Gibbs have recognized that the semiclassical criteria for absorptive bistability can be met using a single atom, but note that quantum fluctuations will cause rapid switching between semiclassically predicted states, and set a level of ~ 1000 atoms or their statistical equivalent for a resonable hysteresis to be preserved [4]. In this paper, we show that single-atom absorptive optical bistability does exist within a quantum mechanical theory.

The quantum mechanical problem can be solved analytically if strong cavity damping justifies the adiabatic elimination of the cavity mode operators on the time scale of the atomic dynamics. In this *bad-cavity limit*, quantum fluctuations do indeed destroy the semiclassical prediction of bistability [2]; this limit is characterized by very small photon numbers, and the failure of semiclassical theory is to be expected. But the opposite limit, the good cavity limit, is characterized by large photon numbers. Then, although the absorber is a single atom, the semiclassical prediction might well be correct. In the *good-cavity limit*, the cavity oscillator amplitude changes slowly on the time scale of the atomic fluctuations. The single-atom absorption is still noisy, but a slow cavity response will smooth these fluctuations, averaging them to give negligible noise strength under the cavity linewidth. We have used a supercomputer to solve the operator master equation describing the coupled atom and cavity mode for systems in between these quantum and classical limits.

Our numerical approach might be used to set fundamental limits for a reliable miniaturized optical switch. However, practical concerns about distinguishing states in the presence of shot noise can be addressed in less sophisticated models based on photon number rate equations. Given the large margins of safety that would be required for a practical device, a rule of thumb like the 1000 quanta switching energy suggested by McCall and Gibbs [4] should be insensitive to the subtleties omitted by such models. On the other hand, these subtleties are essential to more profound questions concerning the interface between the quantum and classical worlds. Our system is an elementary example of a *quantum dissipative system*; not just a quantum system with dissipation, but a quantized nonlinear dynamical system in which dissipation plays a central role in establishing macroscopic states, and in the bifurcation of macroscopic states, far from thermal equilibrium. How do these states "dissolve" into the quantum fluctuations? Where do classical statistical analogies break down? How does quantum probability re-

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As a result of delays in my office, this paper, which was originally scheduled for the July 1988 Special Issue, has been delayed.—W. Streifer, *Editor*.

Single-Atom Cavity-Enhanced Absorption I: Photon Statistics in the Bad-Cavity Limit

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Abstract—The photon statistics of the transmitted light from a driven cavity containing a single resonant two-level atom are studied in the bad-cavity limit. For weak driving fields, the second-order intensity correlation function shows novel nonclassical behavior due to the interference of the driving field and forward reradiation from the atom. This behavior is related to squeezing in the cavity transmission. A physical interpretation is given in terms of the reduced quantum state of the coupled atom-field system following photodetection.

I. Introduction

THE interaction of a single atom with electromagnetic radiation may be altered by confining that radiation inside a cavity. The study reported in this paper differs from most recent work on atom-field interactions inside cavities in two respects. First, to realize large single-mode coupling constants, most recent work uses Rydberg atoms and microwave cavities [1]-[3]. This paper is concerned with photon statistics and envisages experiments at optical frequencies where photon counting measurements are possible. Second, leaving aside cavity-enhanced and -inhibited spontaneous emission, interest in dynamical effects has focused on the Jaynes-Cummings interaction Hamiltonian [4]; specifically, dissipation has been considered only as a perturbation on conservative dynamics to account for a small but finite cavity Q [5]-[8]. In this work, dissipation plays a central role. We study an extension of the Jaynes-Cummings model which includes cavity loss through finite reflectivity mirrors and spontaneous emission to modes other than the privileged cavity mode.

The feasibility of measurements at optical frequencies has been demonstrated in a number of experiments. Recently, cavity-enhanced and -inhibited spontaneous emission were observed at optical frequencies [9], [10]. Also, the single-quantum (weak-field) frequency splitting for a cavity mode coupled to two-level atoms was observed by Brecha *et al.* [11] and used to generate squeezed light, with observed noise reductions of 30 percent relative to the vacuum level [12].

Together, dissipation, a driving field, and the Jaynes-Cummings interaction define what is perhaps the most elementary model for a quantized dissipative dynamical system: a driven damped harmonic oscillator coupled to a damped two-level atom. With the driving field, oscillator, and atom all on resonance, the behavior of this system will be governed by four parameters: the complex driving field amplitude $\mathcal{E}$, the atom-field coupling constant g, and damping rates γ and κ for the atom and field, respectively. One damping rate simply scales the time; therefore, there are really only three parameters. These are conveniently taken as the dimensionless parameters $Y = \mathcal{E}/\kappa\sqrt{n_s}$, $C = g^2/\kappa\gamma$, and $n_s = \gamma^2/8g^2$ or, alternatively, Y, C, and $\mu = 2\kappa/\gamma$. Here n_s is the number of photons in the cavity mode required to saturate the atom (the saturation photon number), and C is the single-atom version of the so-called cooperativity parameter in the theory of optical bistability [13], [14].

For limiting values of the parameters, this model contains many of the effects familiar from recent work: cavity-enhanced spontaneous emission [10], [15]-[21], single-quantum-level splitting [11], [12], [22]-[26], and the collapse and revival of Rabi oscillations [27]-[37]. It also provides an interface to a broader class of phenomena. Our model introduces a flux of energy through the atom-field system. The system then evolves to a nonequilibrium stationary state. Transient behavior will show features associated with the effects mentioned above. In addition, the quantum-statistical properties of the asymptotic state are of interest. Dissipative systems in classical nonlinear dynamics show a remarkable diversity of behavior in the asymptotic limit, including bifurcations, stochastic switching between states, self-oscillation, and chaos [38]. Simple quantum dissipative systems may be expected to display equally diverse phenomena, with the added interest that they may exhibit uniquely quantum-mechanical statistical properties.

A semiclassical factorization assumption reduces our system to the familiar model for absorptive optical bistability [13], [14], [38]-[41]. Then, only the two parameters Y and C are needed to classify the stationary states. (n_s or, more appropriately, $\mu = 1/4Cn_s$, still distinguishes systems according to their dynamical behavior.) Of course, such a factorization is generally a bad approximation. In the quantized theory, n_s determines the "size" of the system. It changes in proportion to the mode volume and scales the energy inside the cavity. Thus, cavity geometries with the same energy density at the site of the atom, but different numbers of photons in the cavity, are possible; identical semiclassical states can be constructed

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Quantum Noise in the Parametric Oscillator: From Squeezed States to Coherent-State Superpositions

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We compare the nonclassical states of light produced by a parametric oscillator for quantum noise of different strengths. Increasing noise strength brings a transition from a slightly perturbed classical state showing squeezing to a superposition of coherent states. We use the positive-P representation to illustrate the roles of quantum noise, quantum coherence, nonlinearity, and dissipation in this simple quantum dynamical system.

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Recent experiments producing squeezed light have added a new nonclassical light source to the few available, thereby rekindling interest in nonclassical states of the electromagnetic field. The degenerate parametric oscillator has played a central role in studies of squeezing; its Hamiltonian is intimately related to the infinitesimal generator of squeezed states. Moreover, it enjoys the distinction of having produced the greatest amount of squeezing observed to date.[1,2]

In a sense, however, parametric oscillators that produce squeezed light are almost classical; they are classical systems driven by a *very small* quantum noise —"small" in the sense (mathematically) that a linearized treatment of the quantum dynamics is valid, and (physically) that many photons are needed to probe the system's nonlinearity. In this Letter we present a treatment of the degenerate parametric oscillator valid for quantum noise of arbitrary strength. Our approach is based upon the positive-P representation. We find an analytic solution for the steady-state positive-P function. This solution is a function of two phase-space variables; one variable is the "classical" field amplitude of semiclassical nonlinear optics; the other is a "nonclassical" variable needed to represent *superpositions* of coherent states. When the positive-P function is plotted in three dimensions the role of the nonclassical variable can be clearly visualized. Distinct pictures emerge for the limiting regimes of essentially classical behavior and predominantly quantum behavior. This distinction is drawn from the novel feature that the quantum dynamics is naturally confined to a *bounded* manifold in phase space; the extent to which the noise has sufficient strength to probe the boundary provides a measure of the deviation from a classical state. This bounded manifold provides a beautiful illustration of the subtle way in which recently reported anomalies in stochastic simulations based on the positive-P representation may be resolved.[3]

The degenerate parametric oscillator is modeled by two quantized field modes, with frequencies ω and 2ω, interacting via a $\chi^{(2)}$ susceptibility inside an optical cavity. Both modes are resonant with the cavity and experience linear loss. The cavity is excited by a classical pump field with frequency 2ω. The microscopic Hamiltonian takes the form

$$H = i\hbar\tfrac{1}{2}\bar{g}(\hat{a}^{\dagger 2}\hat{b} - \hat{a}^2\hat{b}^\dagger) + i\hbar\mathcal{E}(\hat{b}^\dagger - \hat{b}) + H_{\text{loss}}, \quad (1)$$

where $\hat{a}$ and $\hat{a}^\dagger$, and $\hat{b}$ and $\hat{b}^\dagger$, are annihilation and creation operators in the interaction picture; $\bar{g}$ is the mode-mode coupling constant; $\mathcal{E}$ is the intracavity pump-field amplitude; and H_{loss} describes losses in the nonlinear crystal and at the cavity mirrors.

This nonlinear quantum-mechanical problem can be mapped by an appropriate phase-space representation into a classical stochastic process. The familiar Glauber-Sudarshan P representation gives a Fokker-Planck equation without positive-definite diffusion. This difficulty can be overcome with the positive-P representation. With mode $\hat{b}$ adiabatically eliminated we obtain the following set of Ito stochastic differential equations for the complex amplitude[4] of mode $\hat{a}$:

$$\begin{aligned} d\alpha &= [-\alpha - \alpha_*(\lambda - \alpha^2)]d\tau + g(\lambda - \alpha^2)^{1/2}dW_1, \\ d\alpha_* &= [-\alpha_* + \alpha(\lambda - \alpha_*^2)]d\tau + g(\lambda - \alpha_*^2)^{1/2}dW_2, \end{aligned} \quad (2)$$

where dW_1 and dW_2 are independent Wiener increments, τ is measured in cavity lifetimes (γ_a^{-1}), $g = \bar{g}/(2\gamma_a\gamma_b)^{1/2}$, and λ is a dimensionless measure of the pump-field amplitude scaled to give the threshold condition $\lambda = 1$; γ_a and γ_b are decay rates for the cavity fields. The complex variables α and α_* are associated with operators $\hat{a}$ and $\hat{a}^\dagger$, respectively. Stochastic averages of α and α_* give the operator averages $g\langle\hat{a}\rangle$ and $g\langle\hat{a}^\dagger\rangle$. In the Glauber-Sudarshan representation α and α_* are complex conjugates. In the positive-P representation they are not, although they must be so in the mean. More generally, normally ordered averages of quantum operators are calculated from the positive-P function, $P(\alpha, \alpha_*)$, with

$$\langle\hat{a}^{\dagger n}\hat{a}^m\rangle = g^{-(n+m)}\int d^2\alpha \int d^2\alpha_*\, \alpha_*^n \alpha^m P(\alpha,\alpha_*). \quad (3)$$

Equations (2) describe trajectories in a four-dimen-

Squeezed States

Squeezed-state generation for two-level atoms in a spatially varying field mode

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A spatially varying field mode is included in calculating the squeezing effect for a system of two-level atoms in the good-cavity limit. Two examples of a Gaussian mode field in a ring cavity and a plane-wave field in a standing-wave interferometer are used to demonstrate the quite general method. In qualitative terms, the squeezing predicted for plane waves is preserved. However, for a given value of atomic cooperativity parameter C, there is a degradation in squeezing because of the spatially varying field structure.

Squeezed-state generation in an atomic vapor interacting with a cavity field mode has been intensively discussed both experimentally[1–3] and theoretically[1,4–8] in recent years. Some experiments[1,2] have been quite successful in observing squeezing effects at a respectable level. Most theories describing these systems take the cavity field as a plane wave for simplicity. This makes it hard for experiments to be compared quantitatively with the theory. In this paper we illustrate how the spatially varying field mode alters squeezing in a simple system consisting of two-level atoms interacting with a single mode of a high-finesse cavity. We use the general formulas developed earlier in Ref. 9. Two examples, a Gaussian mode in a ring cavity and a plane wave in a standing-wave cavity, are given to illustrate the effect of field variations. Figures and discussions are given for each case and compared with the existing plane-wave theory.[5]

Following Ref. 9, we consider a single, quantized, spatially varying cavity mode, interacting with a collection of N homogeneously broadened two-level atoms that are driven by a coherent classical field of amplitude ϵ. The master equation for the density operator $\hat{\rho}$ of the atom–field system following from the Hamiltonian in the electric-dipole, rotating-wave and Markovian approximations can be written as[10–12]

$$\begin{aligned}\frac{\partial\hat{\rho}}{\partial t} &= \frac{1}{i\hbar}\left\{\hbar\omega_c[\hat{a}^\dagger\hat{a}, \hat{\rho}] + \frac{1}{2}\hbar\omega_a\sum_{\mu=1}^{N}[\hat{\sigma}_\mu{}^z, \hat{\rho}]\right\}\\ &+ \sum_{\mu=1}^{N}\{g_\mu\exp(-i\mathbf{k}\cdot\mathbf{r}_\mu)[\hat{a}^\dagger\hat{\sigma}_\mu{}^-, \hat{\rho}] - g_\mu\exp(i\mathbf{k}\cdot\mathbf{r}_\mu)\\ &\times[\hat{a}\hat{\sigma}_\mu{}^+, \hat{\rho}]\} + \sum_{\mu=1}^{N}\{(\tfrac{1}{2}\gamma_\parallel)([\hat{\sigma}_\mu{}^-\hat{\rho}, \hat{\sigma}_\mu{}^+] + [\hat{\sigma}_\mu{}^-, \hat{\rho}\hat{\sigma}_\mu{}^+])\\ &+ (\tfrac{1}{4}\gamma_P)([\hat{\sigma}_\mu{}^z\hat{\rho}, \hat{\sigma}_\mu{}^z] + [\hat{\sigma}_\mu{}^z, \hat{\rho}\hat{\sigma}_\mu{}^z])\} + \kappa\{[\hat{a}\hat{\rho}, \hat{a}^\dagger]\\ &+ [\hat{a}, \hat{\rho}\hat{a}^\dagger]\} + \kappa\{\epsilon\exp(-i\omega_l t)[\hat{a}^\dagger, \hat{\rho}] - \epsilon^*\exp(i\omega_l t)[\hat{a}, \hat{\rho}]\}. \quad (1)\end{aligned}$$

The operators $\hat{a}^\dagger$ and $\hat{a}$ are the single-mode creation and annihilation operators, while $\hat{\sigma}_\mu{}^z$ and $\hat{\sigma}_\mu{}^\pm$ are the Pauli atomic operators, ω_c is the cavity resonance frequency, ω_a is the atomic resonance frequency, and ω_l is the driving-field frequency. g_μ is the coupling coefficient between the cavity field mode and an atom at position $\mathbf{r}_\mu$ and is given in terms of the normalized mode function $U(\mathbf{r}_\mu)$ by[13]

$$g_\mu = \left(\frac{\mu^2\omega_c}{2\hbar\epsilon_0}\right)^{1/2}|U(\mathbf{r}_\mu)| \equiv g_0|U(\mathbf{r}_\mu)|. \quad (2)$$

κ is the cavity damping rate, $\gamma_\parallel$ is the Einstein A coefficient for spontaneous emission, and γ_p is the rate of collision-induced phase decay, so that the total rate of decay of the atomic polarization $\gamma_\perp$ is given by $\gamma_\perp = \gamma_\parallel/2 + \gamma_p$. The thermal photon numbers in both the atom and cavity-mode reservoirs are set to zero.

We divide the cavity mode into M small sections, according to the mode structure, so that the field can be viewed as effectively constant across each section. The sections are still assumed to be large enough so that the number of atoms in the jth section $N_j \gg 1$ for all j. We checked this assumption for realistic experimental situations with different mode structures and found that it can be reasonably well satisfied. Following Refs. 9–11, we transform the operator master equation (1) into a c-number generalized Fokker–Planck equation in the positive P representation.[11,14,15] Owing to the condition $N_j \gg 1$ for each section, we can truncate this generalized Fokker–Planck equation to second order and write corresponding Ito stochastic differential equations.[16,17] For a high-Q cavity ($\gamma_\perp, \gamma_\parallel \gg \kappa$), we then adiabatically eliminate atomic variables to arrive at equations for field variables alone.

By setting the derivatives to zero and neglecting the fluctuations in the resulting stochastic differential equations for the field, we find that the steady-state solution satisfies

$$Y = X\left|1 + i\phi + \frac{2(1 - i\delta)}{1 + \delta^2}\cdot\frac{C}{s}\sum_{j=1}^{M}\frac{|U(\mathbf{r}_j)|^2\Delta V_j}{1 + \dfrac{V_{\text{eff}}}{s}X\dfrac{|U(\mathbf{r}_j)|^2}{1 + \delta^2}}\right|^2. \quad (3)$$

Squeezed States

Spectrum of squeezing and photocurrent shot noise: a normally ordered treatment

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The detection of squeezed optical fields generated by intracavity nonlinear-optical interactions is described. The relationship between the quantum-statistical properties of the cavity mode and those of the field at a detector placed outside the cavity is discussed. The detected field is composed of a source field from the cavity plus a free external field. The free field couples weakly to the cavity so that it is correlated with the source field. The photocurrent spectrum in optical homodyne detection is calculated, and the spectrum of squeezing is defined. This spectrum can be calculated entirely in terms of intracavity fields without requiring knowledge of the correlations between source and free-field contributions in the cavity output. This follows from an explicitly normally ordered, time-ordered treatment of the photodetection problem. Contrasting earlier treatments of photodetection for squeezed fields, in a normally ordered approach, shot noise arises naturally from the self-correlation of photocurrent pulses. The derived spectrum is converted into nonnormally ordered, non-time-ordered form to recover the results of these earlier treatments and their interpretation of shot noise in terms of local-oscillator quantum noise, signal quantum noise, and detector-efficiency quantum noise.

1. INTRODUCTION

Observations of squeezing have been made in a variety of systems.[1-6] A number of these systems use an optical cavity to enhance the coupling strength in some nonlinear optical interaction. The interaction that generates squeezed light takes place inside the cavity between one or more quantized cavity modes. The cavity output provides the signal field for a balanced homodyne-detection scheme, which analyzes the difference signal between photocurrents from two photodiode detectors. Squeezing is observed over some finite bandwidth as a phase-sensitive reduction of the photocurrent noise below the shot noise level obtained with the cavity output blocked. The canonical system of this type is the degenerate parametric oscillator—an intracavity version of the degenerate parametric amplifier, which occupies a central position in discussions of squeezing, since, in the simplest approximation, the amplified subharmonic evolves under the action of the squeeze operator itself.[7] The degenerate parametric oscillator was recently brought to the forefront as a squeeze generator by the work of Wu *et al.*,[3] who achieved 60% noise reduction.

The example of the parametric oscillator uncovered some important issues in the theoretical analysis of squeezing for intracavity systems. In a high-finesse cavity and in the undepleted pump approximation, the interaction that underlies squeezing can be formulated in terms of a single quantized mode for the subharmonic field. One route to a single-mode description for this system is provided by the operator master-equation methods. It was something of a surprise when the first master-equation treatment of the degenerate parametric oscillator could produce, at best, only 50% squeezing in the intracavity field.[8] The surprise was soundly based. Soon an alternative calculation predicted the possibility for perfect squeezing at the oscillator threshold in a cavity with a single output port.[9]

For all practical purposes the discrepancy in these results has been resolved. Greater than 50% squeezing was observed with a degenerate parametric oscillator,[3] and theoretical results in quantitative agreement with experiment have been obtained.[10] The resolution comes with the recognition that two traps are laid for any calculation that focuses on the cavity mode alone, as in the cited master-equation calculation.[8] First, we should strictly speak of a cavity *quasi-mode*, not a cavity mode. If there is to be an output, the cavity must have at least one partially transmitting mirror, and its modes then acquire a linewidth; these are *quasi-monochromatic*. Thus one might well ask for the frequency decomposition of the squeezing. In a single- (quasi-)mode formulation, this is a question that we are tempted to overlook. Of course, there is nothing new here. Single-mode quantum theories of the laser are widely discussed and not limited in their ability to describe the laser linewidth.[11]

The second trap lies in any loss from the cavity that is not coupled into the detected output beam. Such loss occurs when a cavity has two output ports and light from just one of them is collected by the detection system. This loss alters the relationship between detected photon statistics and photon statistics calculated for the intracavity mode itself. In photoelectron-counting measurements it translates into a random deletion of potential photoelectric counts, which tends to convert a potentially correlated photoelectron-counting sequence into a random counting sequence. This problem is also familiar. An analogy can be drawn between a cavity emitting into detected and undetected output channels and an atom fluorescing into collected and uncollected solid angles. The difficulty that imperfect collection efficiency brings to the observation of sub-Poissonian statistics and squeezing in resonance fluorescence is well documented.[12]

Although the importance of avoiding these traps is now recognized and in practical terms the early problems are

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Resonance fluorescence in a squeezed vacuum

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Abstract. Fluorescence from a coherently driven two-level atom that is damped by a squeezed vacuum is studied. We show that the mean atomic polarization depends on the relative phases of the squeezed vacuum and the coherent driving field. The fluorescent spectrum is calculated and shows several modifications over the spectrum for normal resonance fluorescence. In particular, the central peak of the Mollow triplet has a linewidth that depends on the phase of the driving field. For strong squeezing this peak can either be much narrower or much broader than the natural linewidth of the atom.

1. Introduction

With the recent successful generation of squeezed states of light, interest now turns to possible applications of squeezed light. Squeezed states are quantum states of the electromagnetic field with fewer fluctuations in one quadrature phase than the vacuum [1, 2]. The generation of squeezed states has been reported in four-wave mixing in atomic vapours [3, 4] and optical fibres [5], optical parametric oscillation [6] and optical bistability [7]. In the parametric oscillator a reduction in fluctuations of 60 per cent below the vacuum level has been achieved.

A source of broadband squeezed light may be used to modify the spectroscopic properties of an atom. Gardiner [8] has considered the behaviour of a two-level atom that is damped by its interaction with a squeezed vacuum (squeezed white noise) rather than the normal vacuum field. He showed that the decay of the atomic dipole depends on the phase of the squeezing. The two quadratures of the atomic polarization are damped at different rates, one exhibiting an enhanced decay rate and the other a reduced decay rate compared with the normal radiative decay of the atom. In this paper we wish to consider further applications to atomic spectroscopy. We add a coherent driving field to the problem treated by Gardiner and study resonance fluorescence for an atom that is damped via interaction with a squeezed vacuum.

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Resonance Fluorescence from an Atom in a Squeezed Vacuum

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The fluorescent spectrum for a two-level atom which is damped by a squeezed vacuum shows striking differences from the spectrum for ordinary resonance fluorescence. For strong coherent driving fields the Mollow triplet depends on the relative phase of the driving field and the squeezed vacuum field. The central peak may have either subnatural linewidth or supernatural linewidth depending on this phase. The mean atomic polarization also shows a phase sensitivity.

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The generation of broad band squeezed light has recently been reported using four-wave mixing in atomic vapors[1] and optical fibers,[2] optical parametric oscillation,[3] and optical bistability.[4] In the parametric oscillator a reduction of fluctuations by 60% from the normal vacuum level has been achieved. In this Letter we investigate the spectroscopic properties of an atom interacting with a broad-band squeezed vacuum field.[5,6] Gardiner has considered the radiative decay of a two-level atom interacting with such a squeezed vacuum.[7] He showed that the two polarization quadratures are damped at different rates—one at an enhanced rate and the other at a reduced rate compared to normal radiative decay. We analyze resonance fluorescence from a driven atom which is damped by a squeezed vacuum. Certain atomic properties such as the steady-state atomic polarization, saturation intensity, and fluorescent spectrum, are now phase dependent.

The Hamiltonian describing the interaction of a two-level atom with the quantized multimode radiation field and a classical driving field is given in the electric-dipole and rotating-wave approximations by

$$H=\tfrac{1}{2}\hbar\omega_A\sigma_z-(\mu E e^{-i\omega_L t}\sigma_+ +\mu^* E^* e^{i\omega_L t}\sigma_-)+H_{\text{rad}}+\hbar(\sigma_+\Gamma+\sigma_-\Gamma^\dagger), \tag{1}$$

where ω_A is the atomic resonance frequency, σ_+, σ_-, and σ_z are pseudospin operators for the atom, and μ is the atomic dipole moment; H_{rad} is the free Hamiltonian for the quantized radiation field, Γ and $\Gamma^\dagger$ are operators defined in terms of the positive- and negative-frequency components of this field, respectively, and E is the amplitude of the coherent driving field with frequency ω_L. The normal treatment of resonance fluorescence takes the quantized radiation field in the usual vacuum state. We assume that it is in a broad-band squeezed vacuum state centered about the frequency ω_L. We assume that *all* of the modes coupling to the atom are squeezed so there will be not spontaneous emission into unsqueezed vacuum modes, and that the bandwidth of the squeezing is sufficiently broad that the squeezed vacuum appears as δ-correlated squeezed white noise to the atom. Then correlation functions for Γ and $\Gamma^\dagger$ can be written in the form[8]

$$\begin{aligned}&\langle\Gamma^\dagger(t)\Gamma(t')\rangle=\gamma N\delta(t-t'),\quad \langle\Gamma(t)\Gamma^\dagger(t')\rangle=\gamma(N+1)\delta(t-t'),\\ &\langle\Gamma(t)\Gamma(t')\rangle=\gamma M e^{-2i\omega_L t}\delta(t-t'),\quad \langle\Gamma^\dagger(t)\Gamma^\dagger(t')\rangle=\gamma M^* e^{2i\omega_L t}\delta(t-t').\end{aligned} \tag{2}$$

Here γ is the atomic decay rate for spontaneous emission into the *unsqueezed* vacuum, and N and M are parameters which charactize the squeezing, with $|M|^2\le N(N+1)$, where the equality holds for a minimum uncertainty squeezed state. The variances in the quadrature phases of the squeezed field at the site of the atom are $V(X_\theta)=\frac{1}{2}[N+|M|\cos(\theta-\phi)+\frac{1}{2}]$, where $M=|M|\times e^{i\phi}$; the phase ϕ will depend on details of the scheme used to generate the squeezed vacuum. For a highly squeezed ($N\gg 1$) minimum-uncertainty state the variances in the maximally squeezed quadrature, $\theta=\phi$, and the out-of-phase quadrature, $\theta=\phi+\pi$, are $V(X_\phi)\approx N$, and $V(X_{\phi+\pi})\approx 1/16N$.

FJJ3 Excitation of wave packets

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The large transform bandwidth of a picosecond or subpicosecond laser pulse can simultaneously and coherently excite many electronic states of an atom or molecule. The resulting superposition state has a number of properties very different from those of an ordinary energy eigenstate.

Normal excitation with one or two photons produces a state which has a superposition of different principal quantum numbers but identical angular momentum quantum numbers. The resulting wave packet is a shell localized in r but not in θ and Φ. The shell oscillates radially with a period just equal to the period of a classical electron in a Kepler orbit with the same energy as the average energy of the wave packet states. The wave packet is initially localized to give an uncertainty product approximately equal to the Heisenberg minimum value. Later the wave packet spreads but reforms with nearly the same uncertainty product.

If an rf field is used to dress the Rydberg states before the laser pulse is applied, the atom can be excited to a state which is localized in θ and Φ rather than in r. This state is a linear combination of states all with the same principal quantum number but various angular momentum quantum numbers. These wave packets are localized to a classical Kepler orbit with a definite orientation in space. For an alkali atom, the core polarization causes a torque on the wave packet which produces a precession in the orientation of the orbit. The precession is exactly that predicted in classical theory when an r^{-4} perturbation is applied to a particle in a Kepler orbit. This wave packet also spreads and reforms.

We describe other situations in which the wave packet is localized in all three coordinates and thus is very similar to a classical electron moving in a Kepler orbit. Finally, we discuss experimental implications of this work for electrons in Rydberg states as well as other quasicontinua.

(Invited paper, 25 min)

Friday 1 May 1987 ROOM 305 — AFTERNOON FKK

3:00 PM Quantum Statistics: 2

Leonard Mandel, University of Rochester, Presider

FKK1 Interference experiments with nonclassical light

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Nonclassical states of the light, e.g., single-photon states[1] and squeezed states,[2] are now available in the laboratory; these states exhibit some properties one cannot account for using a classical wave model for light. One main issue involved in the realization of nonclassical states is their use in interference experiments. For example, single-photon interference has a great historical importance in basic concepts of quantum mechanics. Actually, wave/particle duality for the photon can be demonstrated straightforwardly in a single-photon interference experiment. On the other hand, high-precision interferometry was the first motivation for research on squeezed states. The aim is to improve on the shot noise limit, which limits the accuracy of very small phase shift measurements.[3] A squeezed-state interferometer design is described in a related paper.[4]

Here we present an experiment involving a single-photon state of light. Atoms in an atomic beam were excited in the upper level of an atomic cascade, and we used fast counting techniques to isolate the emission of a single atom while it is in the intermediate level of the cascade. The nonclassical character of the emitted light was checked very simply, using the fact that a single photon is not split by a beam splitter. The probability for coincidence counts between both sides of the beam splitter was found to be 10 times smaller than the lower limit allowed by a classical wave model for light.

We then used the same source and detection scheme in an interference experiment; the visibility of the fringes was found to be >98% over the range of coincidence probabilities achieved. Obviously, in that case one cannot say which path was followed by the photon in the interferometer. This is a straightforward illustration of the wave/particle duality for a single photon.

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FKK2 Transition from superfluorescence to amplified spontaneous emission

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The processes of superfluorescence (SF) and amplified spontaneous emission (ASE) are quite well understood theoretically and have been the subject of extensive experimental investigation. However, relatively little work has been directed toward understanding the nature of the emission process in the intermediate regime between these two limits, that is, the regime in which dephasing occurs rapidly enough to modify the emission process yet is not so rapid as to prevent entirely the establishment of a macroscopic dipole moment.[1-3] Indeed, this intermediate regime is particularly interesting because it has not been clear how the effects of noise should be incorporated into the theory when dephasing processes occur during the formation of the macroscopic dipole moment.

We performed an experiment that displays the nature of the emission process in the regime intermediate between SF and ASE. The emission from a crystal of KCl containing ~2 × 10[18] superoxide ions/cm^3 was studied as a function of temperature over the range of 10–30 K. The homogeneous dephasing rate in KCl:O_2^- scales as the third power of the absolute temperature, and hence by changing the temperature, we are able to change significantly the dephasing rate. Emission at 630 nm was excited by up to 60 μJ of energy in a 30-ps pulse at the fourth harmonic of a Nd:YAG laser, and the time evolution of the emission was recorded using a streak camera. Typical results are shown in Fig. 1 for three different values of the crystal temperature. At 16 K the emission is characteristic of superfluorescence with a short time delay; at 23 K the emission is in the intermediate regime; and at 28 K the emission is characteristic of ASE. As predicted[1,2] the time delay is seen to increase with increased dephasing rate.

In Fig. 2 the measured value of the mean delay time is plotted as a function of the dephasing rate. The dashed line gives the prediction of the first-passage-time theory of Haake *et al.*[2] This theory is in good agreement with the measured values in the limit of small dephasing but predicts a time delay shorter than the observed value in the intermediate region. The solid curve gives the mean time delay predicted by a numerical integration of the Maxwell-Bloch equations. In solving these equations we assumed the presence of a fluctuating input noise field whose amplitude was chosen to yield predictions in agreement with those of conventional theories in the limit of no dephasing. These predictions are in good agreement with the experimental results. We believe that the improvement results from our inclusion of the effects of noise during the emission process. Previous theories included the effect of noise only as an initial condition, which is an adequate approximation only when dephasing is relatively weak.

Summarizing: we studied the cooperative emission process in a regime intermediate between that of SF and that of ASE. We find that in this regime the measured time delay is somewhat larger than that predicted by current theories.

(Invited paper, 25 min)

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3. J. Okada, K. Ikeda, and M. Matsuoka, Opt. Commun. 27, 321 (1978).

FKK3 Distribution of photoelectron separation times and photoelectron counting probabilities for resonance fluorescence

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In recent papers by Cohen-Tannoudji and Dalibard[1] and Zoller *et al.*[2] the statistics of quantum jumps in a three-level atom are analyzed in terms of the joint probability density $W(t+\tau;t)$ for a photon emission at time t followed by the next photon emission at time $t+\tau$. $W(t+\tau;t)$ differs from the second-order correlation function $G^{(2)}(t+\tau;t)$ often used to characterize photon statistics; the latter gives the joint emission probability without the condition that there be no emissions in the interval t to $t+\tau$. After dividing by the density for single-photon emission, $W(t+\tau;t)$ gives the probability density for a delay τ between photon emissions.

We focus on the statistics of photoelectric detection rather than photon emission and treat resonance fluorescence from a single two-level atom as an explicit example. Corresponding to $W(t+\tau;t)$ the quantity $W_\eta(t+\tau;t)$ is defined as the joint probability density for a photoelectric count at time t followed by the next photoelectric count at time $t+\tau$, given a detection efficiency η. A theory of photoelectric detection for resonance fluorescence, allowing arbitrary detection efficiency, is constructed solely from the master equation for the fluorescing atom without reference to the fluo-

Quantum fluctuations for two-level atoms in a high-Q cavity with a spatially varying field mode

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An extension of the quantum theory of the atom-field interaction within a high-finesse resonator is made to include spatial variations of the field mode in the limit of small cavity decay rate. The two particular examples of a Gaussian mode field in a ring cavity and a plane-wave field in a standing-wave interferometer are presented to illustrate the method. Analytic expressions are obtained for the incoherent intensity and photon correlations of the transmitted field. In qualitative terms, effects such as sub-Poissonian photon statistics predicted by the plane-wave theory in a ring cavity are preserved. In either the weak-field or dispersive limit the results of the plane-wave theory in a ring cavity are recovered independent of the form of the spatial dependence of the cavity mode.

I. INTRODUCTION

In recent years several quantum statistical treatments of optical bistability have been developed for the system of homogeneously broadened two-level atoms interacting with a single damped cavity field mode. Extensive reviews of this subject can be found in the work of Lugiato[1] and of Carmichael.[2] In broad outline these theories deal with the coherent coupling of a collection of atoms to a high-finesse interferometer mode. Dissipation enters through the radiative decay of the atoms and the damping of the cavity. Steady-state operation is achieved with external excitation in the form of a coherent driving field. The interplay of the nonlinear deterministic dynamics and the quantum fluctuations about steady state gives rise to a diversity of phenomena. Of particular interest are such nonclassical effects as sub-Poissonian photon statistics, photon antibunching, and squeezing, which arise from the nonclassical nature of the fluctuations.

All of the theories of the quantum processes in optical bistability consider a plane-wave field mode in a ring cavity, which is not realistic in many experimental situations. The purpose of this paper is to extend these quantum statistical theories to include spatial variations of the field mode. Our work follows closely that of Drummond and Walls,[3] hereafter referred to as OBII, and is likewise carried out in the "good-cavity" limit, with cavity decay rate much smaller than either the atomic decay rates $(\gamma_\perp,\gamma_\parallel)$ or the cavity coupling coefficient $(\sqrt{N}g)$. We deal with the nonuniformity of the cavity field by dividing the cavity mode into small sections which are each microscopically large in terms of atomic number to allow truncation of the generalized Fokker-Planck equation, but which are macroscopically small to justify the assumption of constant field amplitude.

In Sec. II we obtain the linearized Fokker-Planck equation, and, from that, expressions for the ratio of incoherent intensity to coherent intensity and for the fourth-order field correlation function which describes sub-Poissonian photon statistics. In Secs. III and IV we present two examples to show how this theory applies to more realistic physical systems, namely, to a Gaussian field inside a ring cavity and to a plane wave inside a standing-wave interferometer. The results are illustrated with a number of figures for comparison with the existing literature on the plane-wave ring cavity. Section V serves as a summary of our findings.

II. MODEL AND LINEARIZED THEORY

We consider a single, quantized, spatially varying cavity mode interacting with a collection of homogeneously broadened two-level atoms. The atoms and the cavity are damped through coupling to reservoirs, and the cavity is driven by a coherent field of amplitude $\mathscr{E}$. We divide the cavity mode into M small sections with $N_j \gg 1$ atoms in the jth sector. Each section is assumed to be sufficiently small so that we can view the field as effectively constant across it. By following a procedure similar to that in OBII, using the electric dipole, rotating-wave, and Markovian[4] approximations, we find the quantum master equation can be written as

$$\frac{d\hat{\rho}}{dt}=\frac{1}{i\hbar}\left[\hbar\omega_c[\hat{a}^{\dagger}\hat{a},\hat{\rho}]+\tfrac{1}{2}\hbar\omega_a\sum_{j=1}^{M}[\hat{J}^{z}_{j},\hat{\rho}]\right]+\sum_{j=1}^{M}(g_j[\hat{a}^{\dagger}\hat{J}^{-}_{j},\hat{\rho}]-g_j[\hat{a}\hat{J}^{+}_{j},\hat{\rho}])$$
$$+\sum_{j=1}^{M}\left[(\tfrac{1}{2}\gamma_{\parallel})\sum_{i=1}^{N_j}([\hat{\sigma}^{-}_{ij}\hat{\rho},\sigma^{+}_{ij}]+[\hat{\sigma}^{-}_{ij},\hat{\rho}\hat{\sigma}^{+}_{ij}])+(\tfrac{1}{4}\gamma_p)\sum_{i=1}^{N_j}([\hat{\sigma}^{z}_{ij}\hat{\rho},\hat{\sigma}^{z}_{ij}]+[\hat{\sigma}^{z}_{ij},\hat{\rho}\hat{\sigma}^{z}_{ij}])\right]$$
$$+\kappa([\hat{a}\hat{\rho},\hat{a}^{\dagger}]+[\hat{a},\hat{\rho}\hat{a}^{\dagger}])+\kappa(\mathscr{E}e^{-i\omega_l t}[\hat{a}^{\dagger},\hat{\rho}]-\mathscr{E}^{*}e^{i\omega_l t}[\hat{a},\hat{\rho}])\;. \tag{1}$$

Journal of the Optical Society of America A – v 3, Issue 13, p38-p38, 1986

TUA6 How to choose a solid squeezer

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Squeezed states of light with reduced quantum noise in one quadrature of the light field can be generated by parametric processes in atomic or solid systems. The solid systems, in principle, have the advantage that the ratio of the nonlinear (lossless) interaction to the losses in the solid required to generate the squeezed state can be much larger than the corresponding ratio for an atomic system. In the atomic system, losses are dominated by the Lorentzian absorption tail, decreasing as the inverse square of the frequency shift from resonance. Absorption losses in solids can decrease exponentially (or even faster) at frequencies below an absorption band or an exciton feature while the nonresonant nonlinear response decreases slowly, often with a power-law dependence. We concentrate on the third-order susceptibility for a broad class of semiconductors (e.g., GaAs) and organic materials (e.g., polydiactylene) where losses can be very low. The ratio of nonlinearity to loss required for significant squeezing of light noise is in the same range as that required for a good optical switch. A nonlinear phase shift of π is required while losses are reduced so that only a small fraction of the incident light is absorbed. Several solid systems show promise for obtaining large squeezing for photon energies just below the exciton band-edge feature. Enhanced nonlinearity is also obtained at the threshold energy for two-photon absorption. Present crystal growth technologies can decrease losses due to impurities into a very optimistic regime for squeezed state generation. **(12 min)**

TUA7 Conversion of Poisson photons into sub-Poisson photons by the action of electron feedback

FEDERICO CAPASSO, AT&T Bell Laboratories, Murray Hill, NJ 07974; M. C. TEICH, Columbia U., Radiation Laboratory, New York, NY 10027.

Poisson photons may be converted into sub-Poisson photons by the action of an electron current configured in an external feedback loop. The generation mechanism involves single-photon transitions so that the source can be made arbitrarily sub-Poisson. Unlike previous schemes, nonlinear optics is not invoked. A useful configuration involves a photon emitter illuminating a detector/source combination in a closed-loop system. Two solid-state implementations of the detector/source are suggested. One makes use of electronic dipole transitions between the energy levels of a quantum-well heterostructure; the other operates by electrons impact-exciting electroluminescent centers. **(12 min)**

TUA8 Ultrasqueezed light via multifrequency pumping

BONNY L. SCHUMAKER, California Institute of Technology, Physics Department, Pasadena, CA 91125.

The fundamental and simplest kind of squeezing (broadband squeezing) is produced by pumping a nonlinear medium at a single frequency. The observables squeezed are the electric-field quadrature phases defined relative to the pump frequency. Their measurement requires homodyning against an optical-frequency (the pump frequency) local oscillator. When the medium is pumped at several different frequencies (equally spaced), different kinds of observables exhibit squeezing. Their measurement requires further homodyning, against a radio-frequency (the pump frequency spacing) local oscillator, and, for four or more pump frequencies, against subsequent lower-frequency local oscillators. For a given total pump power, it is possible to achieve greater squeezing by distributing the pump power over several (at least three) different pump frequencies than by pumping at a single frequency. The drawback is that a more sophisticated detection scheme is required (i.e., further homodyning) to measure the maximum squeezing (less squeezing is produced in more easily measured observables, such as the electric-field quadrature phases). The required further homodyning at rf and lower frequencies is straightforward to accomplish, however, and the advantages of using lower pump powers at each optical frequency can be substantial, especially in mediums such as optical fibers, where large pump powers bring with them substantial added noise sources. **(12 min)**

TUA9 Nonclassical photon statistics in the transmission from a resonant cavity containing a single atom

P. R. RICE, H. J. CARMICHAEL, U. Arkansas, Physics Department, Fayetteville, AR 72701.

We consider an optical cavity containing a single two-level atom driven on resonance by an external laser source.[1] We focus on the limit of weak excitation where the dynamic response of this system is governed by just three parameters: the atom-field coupling constant g, spontaneous emission rate γ for the atom, and cavity decay rate κ. The statistics of the transmitted light are analyzed as a function of g, γ, and κ, in terms of the second-order correlation function $g^{(2)}(\tau)$, and the quadrature variances measured in a homodyne detection scheme. Squeezing and photon antibunching exist over a wide range of parameters. For $\gamma \sim \kappa$, the system can exhibit an oscillatory response, even when the mean intracavity photon number is much less than unity.[2] The oscillations can be understood in terms of the coupling between the free atom-field eigenstates $|0,->$, $|0,+>$, and $|1,->$. The oscillation frequency is determined by the energy level splitting produced between the degenerate one-quantum states $|0,+>$ and $|1,->$ by the atom-field interaction. The second-order correlation function can exhibit a novel nonclassical effect, where, for $g^{(2)}(0) \neq 0$, at some finite delay τ_0, the correlation function dips exactly to zero—$g^{(2)}(\tau_0) = 0$. **(12 min)**

1. H. J. Carmichael, Phys. Rev. Lett. **55**, 2790 (1985).
2. H. J. Carmichael, Phys. Rev. A **33**, 3262 (1986).

NOTES

Tuesday **MORNIN**

21 October 1986 **TU**

OLYMPIC ROOM

8:30 AM **Symposium on Superlattic Infrared Detectors**

L. N. Durvasula, Night Vision and Electr Optics Laboratory, Presider

TUB1 HgTe-CdTe superlattice infrared dete tors

T. C. McGILL, California Institute of Technology, J. Watson, Sr. Laboratory of Applied Physics, Pa adena, CA 91125.

A great deal of interest has developed in the u of superlattices as infrared materials. The infr red superlattice to be discussed is the one form by laying down repeatedly a layer of $Hg_{x_1}Cd_{1-x_1}$ followed by a layer of $Hg_{x_2}Cd_{1-x_2}Te$ with $x_1 \neq$ To date, most of the research has been for case when $x_1 = 1$ and $x_2 = 0$. The theoreti studies have indicated that superlattices could pr vide an interesting solution to a number of pro lems that exist with the alloy materials. For exa ple, the band gap of the superlattice is adjusted adjusting the thickness of the HgTe and CdTe la ers, in contrast to the alloy, where the relati concentration of the Hg and Cd are used to cont the gap. Particularly in the case of narrow ba gap systems, this form of control seems easier accomplish. In the superlattice, the direct re tionship between the effective mass and the ba gap is broken making possible the suppression tunneling, leakage currents even in very narr band gap materials. These superlattices ha been successfully fabricated. Current research emphasizing the near band gap properties a structural characterization of these superlattic In this presentation, we make a critical exami tion of what we know about these properties the degree to which there is agreement betw theory and experiment. **(Invited paper, 25 m**

TUB2 Strained-layer superlattice detectors a detector concepts

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Strained-layer superlattices (SLSs) made of ternating layers of InAsSb have recently been p posed as alternative III–V materials for infra detector applications in the 8–12-μm wavelen range.[1] The wavelength cutoffs of certain S structures in this material system are theoretic predicted to be extendable to 12 μm through effects of the elastic layer strains in the InAs layers. These III–V materials are expected to more stable than the HgTe-rich HgCdTe alloys to the greater bond strengths of the III–V materi However, the use of superlattices as photovol detector materials introduces new considerati for the design of high efficiency devices. In pa ticular, superlattices typically exhibit minority ca rier diffusion lengths along the growth directi which are less than one-tenth of the in-plane diffu sion lengths.[2] This paper reviews the issues in nt is ultimately limited and the antibunching lost ogether as the amplitude of the coherent carrier reduced to a level comparable to that of the field ctuations (which are set by the system size). ollowing the suggestion of Levenson *et al.*,[2] we vestigate the use of auxiliary cavities for the ntrol of phase and amplitude in the propagation d detection of squeezed states of light. While r a small degree of squeezing such filter cavities perate in a straightforward fashion, a mixing of ld quadratures can occur as the degree of queezing increases leading to a loss of nonclassi- al behavior. **(12 min)**

1. C. W. Gardiner and M. J. Collett, Phys. Rev. A **31**, 3761 (1985).
2. M. D. Levenson, R. M. Shelby, and S. H. Perlmutter, Opt. Lett. **10**, 514 (1985).

UJ5 Squeezing for intracavity generated light the master equation approach

J. CARMICHAEL, U. Arkansas, Physics Department, Fayetteville, AR 72701.

Master equation methods provide useful tools r analyzing the statistics of quantum optical sysms. They focus attention on a simple system of terest (a single cavity mode, for example), acounting for its interaction with its environment in arameters describing decay rates and environental fluctuations. Intracavity systems for generating squeezed light are prime candidates for nalysis by these methods. It has been shown, owever, that it is important in such analysis to istinguish between the statistics of the intracavity eld and the statistics of the field generated outide the cavity by transmission through the cavity mirrors.[1,2] It is desirable to perform calculations with the single mode inside the cavity. Then the statistics of the multimode field outside the cavity must be related to those of the internal mode. The situation is analogous to that in resonance fluorescence, where the statistics of the fluorescent field can be calculated in terms of the source dipole operator. In this paper the field outside an optical cavity is expressed in terms of the intracavity oscillator as a source. The measurement of squeezing in the external field is then related to the correlation properties of the cavity source operator. Measurement by photon counting of homodyned light is related to the spectrum of squeezing calculated by Walls and co-workers. **(12 min)**

1. B. Yurke, Phys. Rev. A **29**, 408 (1984).
2. M. J. Collett and C. W. Gardiner, Phys. Rev. A **30**, 1386 (1984).

TUJ6 Squeezing in nondegenerate two-photon two-level media

BARBARA A. CAPRON, DAVID A. HOLM, MURRAY SARGENT III, U. Arizona, Optical Sciences Center, Tucson, AZ 85721.

We recently[1] calculated the amount of squeezing predicted by our quantum theory of multiwave mixing for a one-photon two-level model. Here we use the corresponding theory for the two-photon two-level model,[2] which allows for nonsaturating nondegenerate pump and probe waves and includes dynamic Stark shifts. We simplify the more complicated general model using a fourth-order perturbation limit and find very simple expressions that show even for very weak pumps with no Stark shifts or atomic detuning significant squeezing can be achieved. Much of this squeezing is due to scattering of the conjugate field off the pump-induced two-photon coherence, which does not arise in the one-photon case. In general, Stark shifts induce large regions of significant gain, resulting in instabilities. We also show results for high intensity pumps that require larger atomic detunings to obtain a large amount of squeezing. The low intensities required in the two-photon case should make squeezing easier to observe than in the one-photon case. **(12 min)**

1. D. A. Holm, M. Sargent III, and B. A. Capron, Opt. Lett. **11**, 443 (1986).
2. D. A. Holm and M. Sargent III, Phys Rev. A **33**, 1073 (1986).

TUJ7 Quantum theory of multiwave mixing in two-photon two-level media

DAVID A. HOLM, MURRAY SARGENT III, U. Arizona, Optical Sciences Center, Tucson, AZ 85721.

Multiphoton transitions are of great interest in laser spectroscopy because they permit the study of new physical phenomena and because of their potential applications. Malcuit *et al.*[1] studied two-photon transitions in atomic sodium and showed that for appropriate phase-matching four-wave mixing can overcome the usual spontaneous decay processes. We have recently developed a quantum theory[2] to treat such multiwave mixing processes in two-photon media. Because of the greater complexity of two-photon transitions, many effects arise that are absent in the one-photon case, some of which have important consequences in the generation of squeezed states. For the two-photon two-level model, the field modes have frequencies approximately one-half the frequency difference between the levels. We find that our coefficients differ from the corresponding one-photon theory in that dynamic Stark shifts have a major effect and that the pump-induced coherence between the two levels has a new term in the conjugate coupling terms, which leads to improved squeezing. **(12 min)**

1. M. S. Malcuit, D. J. Gauthier, and R. W. Boyd, Phys. Rev. Lett. **55**, 1086 (1985).
2. D. A. Holm and M. Sargent III, Phys Rev. A **33**, 1073 (1986).

TUJ8 Theory of squeezed light generation in optical fibers

G. J. MILBURN, Australian National U., Canberra, ACT, Australia; M. D. REID, D. F. WALLS, U. Waikato, Physics Department, Hamilton, New Zealand; R. SHELBY, M. D. LEVENSON, IBM Almaden Research Center, 650 Harry Rd., San Jose, CA 95120-8099.

We give a quantum theoretical analysis of squeezed light generation via nondegenerate four-wave mixing in an optical fiber. The medium is modeled by an ensemble of anharmonic oscillators and loss is included. We include the coupling to acoustic phonons which gives rise to guided acoustic wave Brillouin scattering (GAWBS). The GAWBS introduces random noise which destroys the squeezing at room temperatures. By cooling the fiber to temperatures ~2 K the GAWBS is substantially reduced and the observation of squeezing becomes possible. A comparison is made with the recent experimental results of Levenson *et al.*[1,2] A scheme to suppress the GAWBS at room temperatures using a two-frequency pump and difference detection is analyzed and the conditions in which good squeezing results are determined. **(12 min)**

1. M. D. Levenson, R. M. Shelby, A. Aspect, M. Reid, and D. F. Walls, Phys. Rev. A **32**, 1550 (1985).
2. M. D. Levenson, R. M. Shelby, and S. H. Perlmutter, Opt. Lett. **10**, 514 (1985).

TUJ9 Theory of squeezing in low *Q* cavity

A. LANE, M. D. REID, D. F. WALLS, U. Waikato, Physics Department, Hamilton, New Zealand.

A theory calculating the squeezing generated via intracavity four-wave mixing in a two-level atomic medium is presented. Previous theories have assumed a high *Q* cavity such that the atomic linewidth $\gamma_\perp$ is much greater than the cavity linewidth κ and one can adiabatically eliminate atomic variables. We do not make this assumption.

The transmitted squeezing and intensity spectra of the resonant cavity mode are calculated for a range of ratios $\kappa/\gamma_\perp$. While the result for squeezing at the center frequency is unchanged, one finds at high intensities saturating the atoms that significantly better squeezing is obtainable in the wings of the low Q ($\gamma_\perp \ll \kappa$) cavity spectrum. Such squeezing is not possible in the high Q cavity ($\gamma_\perp \ll \kappa$), because the transmitted spectrum has a width determined by κ and is much narrower than the center peak of the atomic fluorescence spectrum. This fluorescence destroys squeezing in the high Q cavity as one approaches saturation.[1] We also calculate the spectrum of squeezing nondegenerate four-wave mixing where four-wave mixing occurs between different cavity modes. **(12 min)**

1. M. D. Reid and D. F. Walls, Phys. Rev. A **31**, 1622 (1985).

Tuesday **AFTERNOON**

21 October 1986 **TUK JOINT**

OLYMPIC ROOM

1:00 PM ANNUAL/ILS Joint Symposium on Ultrahigh Speed Photodetectors

Federico Capasso, AT&T Bell Laboratories, Presider

TUK1 Photoconducting antennas

D. H. AUSTON, M. C. NUSS, P. R. SMITH, AT&T Bell Laboratories, 600 Mountain Ave., Murray Hill, NJ 07974.

Photoconductors have proved to be extremely effective sources of very short electrical pulses.[1] When illuminated by ultrafast optical pulses they have been used to generate electrical transients as fast as 0.5 ps. The frequency spectrum of these pulses extends from dc up to terahertz frequencies, making them potentially useful sources of microwave, millimeter-wave and far-infrared radiation.

Usually, ultrafast photoconductors are mounted in transmission line structures which are used to propagate the signals to a test system or measurement device. An alternative arrangement which we have demonstrated is to enable the photocurrent to radiate directly into free space as an antenna. In our initial experiments[2] the photoconductors had a radiating area that was extremely small compared to the shortest wavelength of the radiated emission. Their radiation properties could be approximately described by a classical Hertzian dipole. A novel feature of these photoconducting Hertzian dipoles is their reciprocal property which enables them to be used equally effectively as receiving antennas as well as transmitting antennas. This permits a complete measurement sys-

Quantum fluctuations in absorptive bistability without adiabatic elimination

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A linearized theory of fluctuations for absorptive bistability based on the positive P representation is developed without adiabatic elimination of the atoms or the field. An analytic expression for the steady-state covariance matrix is derived from which the size of quantum-statistical effects can be estimated without restriction to the good- or the bad-cavity limit. When the atom and field relaxation rates are similar the intensity correlation function of the transmitted light exhibits an oscillatory relaxation associated with vacuum Rabi splitting.

I. INTRODUCTION

The extensive literature on fluctuations in absorptive bistability is restricted almost exclusively to treatments in the good-cavity and bad-cavity limits.[1] These limits simplify the analysis by allowing for the adiabatic elimination of the atoms or the field, thus reducing the dimensions of the mathematical description. Experiments may not find these limits so convenient, however. For example, recent experiments on absorptive bistability using optically prepumped sodium atomic beams have an atomic decay rate just two or three times faster than the cavity decay rate.[2] These experiments achieve good quantitative agreement with the theory for homogeneously broadened two-level atoms, and it now seems feasible to move to measurements of quantum-statistical effects.[1,3-8] Such measurements will require adherence to very restrictive experimental design. Consider photon antibunching as an example.[5-7] Atomic lifetimes are short in the optical regime and the most manageable time scales are then found in the good-cavity limit. However, the predicted effect is very small in a large system. This calls for a small-cavity design where a decay rate only slightly slower than the atomic decay rate is all that can reasonably be achieved. Since design for the smallness of the effect is so critical, factors of two or three in estimating its size are important. It is not sufficient to merely observe that the effect varies inversely as the saturation photon number n_s or the number of interacting atoms N. These are related by $n_s = N/4C\mu$, where C is the bistability parameter and μ is the ratio of cavity and atomic linewidths. They can differ by orders of magnitude and it is necessary to have all of the factors of C and μ in place for an accurate estimate. Existing theories cannot provide this precision for $\mu \sim 1$. These considerations have motivated the present work in which I develop a linearized quantum-statistical theory of absorptive bistability without adiabatically eliminating the atoms or the field.

Aside from providing quantitative precision between the good-cavity and bad-cavity limits, my general treatment reveals one notable new feature which is missed in both of these limits. When the atomic and cavity decay rates are similar the relaxation of fluctuations can be oscillatory for arbitrarily small intensities, and exact resonance of the driving field, cavity, and atoms. These oscillations are displayed in the intensity correlation function of the transmitted light and will give rise to a doublet in the incoherent component of the transmitted spectrum. They arise from the so-called vacuum Rabi splitting,[9,10] where the degenerate first excited state of the composite system of atoms and cavity mode is split by the atom-field interaction. There are no Rabi oscillations in the population inversion, but the normal modes of the coupled field and atomic polarization are moved from resonance with the driving field; hence the oscillations in field characteristics. I give a novel treatment of this effect in terms of a coupled harmonic oscillator model derived using the Schwinger representation. This treatment demonstrates the role played by atomic and cavity decay in vacuum Rabi splitting for the first time.[11]

In the following section I briefly review the model for absorptive bistability and the methods of the positive P representation which are used to obtain a quantum-statistical formulation in terms of a linearized Fokker-Planck equation. In Secs. III and IV I solve for the steady-state covariance matrix and find expressions for the ratio of incoherent and coherent intensities, the second-order correlation function, and the variance of fluctuations in the field quadratures for the transmitted light. Section V discusses the oscillation associated with vacuum Rabi splitting and the coupled oscillator model for this effect. Section VI provides a summary and conclusions.

II. MODEL AND LINEARIZED THEORY OF FLUCTUATIONS

I consider a collection of N homogeneously broadened two-level atoms interacting on resonance with a single quantized ring-cavity mode

$$\mathbf{E}(z,t) = i\hat{\mathbf{e}}(\hbar\omega_0/2\epsilon_0 V_Q)^{1/2}[a(t)e^{ik_0 z} - a^\dagger(t)e^{-ik_0 z}] , \tag{2.1}$$

where $a^\dagger$ and a are creation and annihilation operators for cavity photons, ω_0 is the resonant frequency,

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Quantum Statistics of Small Bistable Systems

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1. Introduction

Quantum-statistical theories of optical bistability were developed soon after the first experimental observation of bistability some ten years ago [1]. Various quantum-statistical effects have been predicted, including: quantum induced transitions between states [2], evidence of atomic collectivity in the transmitted spectrum [3-5], photon antibunching [5,6], and squeezing [7]. The enthusiasm shown in theoretical analysis has not been matched by experiments, however. One reason is surely that these effects are very small in a system of macroscopic size. Generally they scale inversely with the size of the system, as measured by the saturation photon number n_s or the number of atoms N.

Recent experiments by ROSENBERGER et al. [8] closely meet the idealized conditions of homogeneously broadened two-level atoms in a ring cavity assumed by the theory. Moreover, these experiments achieve quantitative agreement with theory for steady-state features, in the absence of quantum noise. In these experiments $n_s \sim 10^3$ and $N \sim 10^4 - 10^5$. These numbers are too large for a quantum-statistical study. The time is right, however, to consider the measurement of quantum-statistical effects in a new generation of smaller systems. This paper reports recent theoretical results which have arisen from a consideration of such experiments.

Results from three lines of inquiry are reported. First, the extensive literature on quantum fluctuations in optical bistability is limited almost exclusively to treatments in the good cavity and the bad cavity limits. These limits simplify theoretical analysis but may not be so convenient for experiments. Existing theories cannot provide an accurate estimate of the size of quantum effects -- photon antibunching for example -- in between the good and bad cavity limits. Since the design for a small effect is so critical, it is not sufficient to simply assume an inverse dependence on n_s or N. These numbers are related by $n_s = N/4C\mu$, where C is the bistability parameter and μ is the ratio of cavity and atomic linewidths. They can differ by orders of magnitude, and it is necessary to have all the factors of C and μ in place for an accurate estimate. I have developed a linearized theory of quantum fluctuations for absorptive bistability without adiabatic elimination of the atoms or the field [9]. The consideration of photon antibunching in a system with similar atomic and cavity decay rates has brought a bonus. The second-order correlation function for the transmitted light shows an oscillatory response which is related to the so-called "vacuum Rabi splitting" [10].

Two other lines of inquiry recognize the fact that quantum-statistical effects are maximized under conditions where linearized theory must eventually break down. For example, near the critical point, or in a very small system of just a few atoms. I present results throughout the critical region from a fully nonlinear theory of absorptive bistability based on a set of stochastic differential equations derived using the positive P-representation [11]. These results provide a comparison with linearized theory for parameters chosen to correspond to a small system which might reasonably be realized in the laboratory -- with $n_s = 10$ and N = 320 for C = 4 and $\mu = 2$. Finally, I present results from a separate analysis which is suited to the smallest possible system -- a single atom in a high Q cavity [12]. The question of bistability in this system is not addressed; I focus on the phenomenon of photon antibunching and ask: What

Off-resonant-mode instabilities in mixed absorptive and dispersive optical bistability

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We analyze instabilities in mixed absorptive and dispersive optical bistability in the rate-equation approximation and the mean-field and good-cavity limits. Our starting point is a set of multimode equations derived from the Maxwell-Bloch equations for ring-cavity boundary conditions. We obtain analytic expressions for the instability conditions. In a plane-wave analysis, we find that a portion of the lower transmission branch can be unstable in addition to the upper-branch instability found in purely absorptive bistability. Also, a new disconnected region of instability can exist on the upper branch. Our analysis becomes particularly simple for equal and opposite cavity detunings and we explore this case in detail. We extend our treatment to include a Gaussian transverse intensity profile and show that the instabilities remain in the presence of Gaussian averaging. We also show that many of the results obtained in the rate-equation approximation hold when the atomic linewidth and atomic decay rate are of the same order.

I. INTRODUCTION

Optical systems are very appropriate for the study of instabilities, self-pulsing, and chaotic behavior. One particular advantage they have over hydrodynamic systems, for example, is that cavities can be used to tailor the mode structure of the optical fields. One can then deal with tractable theories involving just a few modes and still expect good agreement with experiments.

The study of instabilities in optical bistability[1] (OB) was initiated in work by McCall[2] and Bonifacio and Lugiato.[3] The analysis in Ref. 3 is based on the ring-cavity model for absorptive OB, formulated in terms of Maxwell-Bloch equations for a collection of homogeneously broadened two-level atoms interacting with a plane-wave field which satisfies ring-cavity boundary conditions. The incident field is exactly tuned to a cavity resonance—cavity detuning $\Theta=0$—and to the atomic line center—atomic detuning $\Delta=0$. Instability arises in the good-cavity limit, where the cavity linewidth κ is much less than both the atomic linewidth $\gamma_\perp$ and longitudinal relaxation rate $\gamma_\parallel$, provided the nearest nonresonant cavity modes are detuned from the incident laser by less than the Rabi frequency. Along part of the upper branch of the hysteresis cycle, a set of off-resonance cavity modes, symmetrically placed with respect to the resonant mode, are unstable and undamped self-pulsing arises. In the mean-field limit, the pulsation period is of the order of the cavity round-trip time t_R, corresponding to a beat frequency determined by the longitudinal mode spacing in the empty cavity; the mean-field limit is defined with $\alpha L \ll 1$, $T \ll 1$, and $\alpha L/T$ arbitrary, where α is the unsaturated absorption coefficient, L is the length of the atomic sample, and T is the mirror transmission coefficient. Outside the mean-field limit, the pulsing frequency is renormalized by the atom-field interaction, but remains of the order of t_R^{-1}.[4]

For the general case $\Delta\neq 0$, $\Theta\neq 0$, the ring-cavity model of OB is analyzed in Ref. 5 after adiabatic elimination of the polarization (rate-equation approximation). This requires $\gamma_\perp \gg \gamma_\parallel$ and $\gamma_\perp \gg 2\pi/t_R$, where the second inequality requires the free spectral range to be much less than the atomic linewidth. Under these conditions, the ring-cavity model can be formulated as a set of differential difference equations.[5] If the free spectral range is also much smaller than the longitudinal decay rate, $\gamma_\parallel \gg 2\pi/t_R$, the model simplifies to a two-dimensional discrete map. Then, when the steady state becomes unstable, all cavity modes are simultaneously unstable, and spontaneous pulsations arise with a period equal to twice the cavity round-trip time. The first prediction of chaotic behavior in OB was made for this model.[5,6] By suitably varying the incident intensity, the pulsation at twice the round-trip time period doubles to chaos. This behavior was first seen experimentally in a hybrid electro-optic device[7] and more recently has been seen in all-optical systems under transient conditions.[8,9]

The relationship between the results of Refs. 3 and 4 and those of Refs. 5 and 6 has been discussed by Lugiato *et al.*[10] and Carmichael.[11] In the mean-field limit, pulsations of period $2t_R$ arise when the incident field is tuned midway between adjacent cavity resonances. In Ref. 11 it is shown that the eigenvalues of the linearized stability analysis for a cavity tuned to resonance (Refs. 3 and 4) and for a cavity tuned between resonances (Refs. 5 and 6) are related by a simple symmetry. Many other papers have studied these and related instabilities; see especially

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THEORY OF QUANTUM FLUCTUATIONS IN OPTICAL BISTABILITY

H J CARMICHAEL

1. INTRODUCTION

In 1969 Szöke et al. proposed a simple scheme for a "bistable optical element" (Szöke et al., 1969). They suggested that a passive optical resonator containing a saturable absorber could exhibit two different output intensities for the same input intensity. In the same year, the same proposal was recorded independently in a patent by Seidel (1969). McCall also recognized the possibility for absorptive bistability, and published a detailed theory for a "bistable mirror" in 1974 (McCall, 1974). Two years later Gibbs et al. (1976) reported the first experimental observation of optical bistability. Their interpretation of this experiment led to the first description of bistability based on nonlinear dispersion. From these beginnings the interest in optical bistability has grown considerably. Several review works are already available (Bowden et al., 1981; Abraham and Smith, 1982; Bowden et al., 1984; Englund et al., 1984; Lugiato, 1984; Wherrett and Smith, 1984; Gibbs, 1985). Fueling much of the development is the search for a viable optical signal processing technology--the oft-quoted "optical transistor" analogy. Questions of fundamental interest, rather peripheral to this theme, have also been highlighted; optical bistability has been adopted as a paradigm for studies in quantum optics, nonequilibrium statistical mechanics, and nonlinear dynamics.

Increasingly, the field of optical bistability is identified with the development of signal processing capabilities. This is not surprising. This theme unifies a large body of work:

Photon Antibunching and Squeezing for a Single Atom in a Resonant Cavity

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(Received 12 August 1985)

The transmitted light from an optical cavity containing a single two-level atom may show photon antibunching and squeezing. The two effects are closely related and simply understood in terms of the theory of single-atom resonance fluorescence. It follows that corresponding nonclassical effects in optical bistability do not originate in atomic collectivity.

PACS numbers: 42.50.+q, 32.80.−t

The behavior of atoms inside cavities is of central interest to studies in quantum optics. The interaction between electromagnetic radiation and matter takes its simplest form for the single-mode electromagnetic field which can be realized inside a high-Q cavity.[1] In a realistic system dissipation will be present, at least in small measure, entering via a finite cavity Q, and, in an open-cavity geometry, via spontaneous emission into noncavity modes. Spontaneous emission inside a cavity brings its own surprises. The emission rate is inhibited in a cavity with dimensions small compared to the transition wavelength, and enhanced in a resonant cavity.[2-4] The emission spectrum may be double peaked, as a result of Rabi splitting of the degenerate single-quantum state of the atom-field system—the so-called "vacuum Rabi splitting."[5,6] The development of experimental expertise with Rydberg atoms has been an important stimulus for interest in this area, recently providing several demonstrations of quantum-dynamical processes inside cavities.[7-10] Of particular note for the present work is the demonstration by Goy *et al.* of enhanced spontaneous emission.[9] This Letter discusses the behavior of a single two-level atom in a coherently driven resonant cavity. Behavior in this single-atom system is related to quantum-statistical effects in absorptive bistability.

The quantum theory of optical bistability considers the steady-state interaction of a collection of two-level atoms and a driven cavity mode in the presence of dissipation.[11] Bistable switching is just one of the interesting phenomena which this general model can address. A number of the predictions from the quantum theory of optical bistability have little to do with bistability itself, but hold much in common with the aforementioned properties of atoms inside cavities. This relationship has been largely overlooked to date. Vacuum Rabi splitting provides an example. I have recently shown that this effect is contained in the theory of absorptive bistability, where it is evidenced in the intensity correlation function and spectral density of the transmitted light.[12] It occurs at weak incident intensities, well below those required for bistable switching; in fact, it does not even require that the conditions for bistability be met. Photon antibunching and squeezing are predicted for this same parameter regime. The present communication reports the novel effect of photon antibunching and squeezing in the transmitted light from a resonant cavity containing a *single* atom. In the bad-cavity limit this effect may be understood in terms of the theory of single-atom resonance fluorescence, and the enhanced spontaneous emission rate for an atom in a resonant cavity. Photon antibunching and squeezing in optical bistability find their origin in this single-atom effect.

According to the theory of absorptive bistability, at weak incident intensities, the size of photon antibunching for homogeneously broadened two-level atoms in a resonant cavity is characterized by[12,13]

$$g^{(2)}(0)=1-N^{-1}4C\mu(\mu+1)^{-1}2C(2C+1)^{-1}, \tag{1}$$

where N is the number of atoms, $\mu=\kappa/\gamma_\perp$ is the ratio of cavity and atomic linewidths, and $C=Ng^2/2\kappa\gamma_\perp$, with g the atom-field coupling constant; $g^{(2)}(\tau)$ is the normalized second-order correlation function for the transmitted light. The predicted effect is too small to be measured in existing experiments.[14] However, smaller experimental systems seem feasible in which a measurable effect might be obtained. What, in principle, is the maximum attainable effect? Equation (1) is only valid for $N>>1$. This limitation is revealed by the observation that $g^{(2)}(0)$ may be negative for small N—for reasonable values of C and μ. By considering a single-atom system I set a reliable limit for the size of the photon antibunching effect.

I begin from the master equation for a single two-level atom interacting on resonance with a single cavity mode resonantly excited by a classical driving field. If ρ denotes the density operator in a frame rotating at the frequency of the driving field, then

$$\dot\rho=\epsilon[a^\dagger-a,\rho]+g[a^\dagger\sigma_- - a\sigma_+,\rho]+\tfrac{1}{2}\gamma(2\sigma_-\rho\sigma_+-\sigma_+\sigma_-\rho-\rho\sigma_+\sigma_-)+\kappa(2a\rho a^\dagger-a^\dagger a\rho-\rho a^\dagger a), \tag{2}$$

SQUEEZING IN THE DEGENERATE PARAMETRIC OSCILLATOR

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and

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Received 9 May 1985

Squeezing of the intracavity field in a degenerate parametric oscillator is calculated above threshold neglecting pump depletion. Arbitrary squeezing is in principle obtainable. We conclude that above threshold squeezing in the steady-state parametric oscillator is limited primarily by pump depletion rather than degradation by vacuum fluctuations at the mirrors. The squeezing predicted by the present calculation should be observable in an oscillator operated in a transient or pulsed mode.

1. Introduction

The degenerate parametric amplifier has featured at the center of discussions concerning the generation of squeezed light for a number of years. In the simplest single-mode model, without pump depletion, the time evolution operator in a quantized theory is just the unitary operator whose action converts the vacuum into a squeezed state [1]. Since the squeezing obtained depends on pump power and interaction time (crystal length), the advantages to be gained by resonating the fields in an optical cavity have directed attention to the degenerate parametric oscillator as a source of squeezed light. An early calculation by Milburn and Walls using master equation methods and the complex P-representation produced rather pessimistic results [2]. Maximum squeezing by a factor of two was predicted at the threshold for parametric oscillation. In contrast, a calculation by Yurke [3] using rather different methods was more optimistic, predicting the possibility for perfect squeezing at threshold. The discrepancy between these calculations has recently been resolved [4,5] by recognizing the distinction between squeezing in the intracavity field, as calculated by Milburn and Walls, and in a frequency selected component of the output field, as calculated by Yurke [4,5]. It is found that squeezing in the intracavity field is degraded by the feeding of vacuum fluctuations associated with dissipation at the cavity mirrors into the cavity mode. By selecting a narrow band of frequencies in the output from a cavity with one perfect reflector arbitrary squeezing is possible.

A comparison of the models used by Milburn and Walls [2] and Yurke [3] reveals a second potentially important difference. Milburn and Walls include a quantized pump mode, allowing for pump depletion, and their model therefore evolves to a steady state above threshold. Yurke describes the amplifying element by a constant gain and has no pump depletion. His results are only applicable below threshold; above threshold the steady state solution is unstable and any small fluctuation leads to unbounded growth of the subharmonic. The more recent multimode theories of Collett and Gardiner [4] and Gardiner and Savage [5] must similarly be restricted to below threshold. It appears that a treatment above threshold in the undepleted pump approximation has not been carried out. Here transient solutions must be constructed, but this is no difficulty since the equa-

MULTIMODE INSTABILITIES FOR A STANDING-WAVE CAVITY CONTAINING A SATURABLE ABSORBER

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Received 8 June 1984

The stability of a coherently driven standing-wave cavity containing a saturable absorber is analyzed in the mean-field limit. Instabilities for a cavity tuned to resonance and a cavity tuned midway between resonances are compared. With the absorber appropriately positioned in the cavity a one-to-one correspondence exists between optical bistability in a resonant cavity and self-oscillation in a cavity tuned between resonances.

1. Introduction

I have recently shown [1] that the multimode stability analysis for a nonlinear ring cavity exhibits a symmetry which relates optical bistability in a cavity tuned near resonance to self-oscillation in a cavity tuned between resonances. This relationship unifies two classes of multimode instabilities: Bonifacio and Lugiato's self-pulsing instability [2], which arises together with optical bistability in a cavity tuned near resonance, and Ikeda's instability leading to period-doubling and chaos [3], which arises in a cavity tuned between resonances. The purpose of this communication is to present a stability analysis for the nonlinear standing-wave cavity and look for a similar relationship there.

A fundamental difference between the ring and standing-wave cavities is that in the latter stability criteria depend on the length and position of the intracavity medium. Casagrande et al. [4] have published a multimode stability analysis for absorptive bistability in a standing-wave cavity where the absorber fills the cavity. They find nonresonant mode instabilities along the upper branch of the bistability curve for $C \gtrsim 60$. Sargent [5] has calculated the weak-field gain for the first pair of nonresonant modes in absorptive bistability. He finds an instability when the absorber is localized at the ends of the cavity, but no instability with the absorber in the center of the cavity; he suggests that the second pair of nonresonant modes might become unstable in the latter case. He finds no instability when the absorber fills the cavity. Firth [6] has found Ikeda instabilities in a standing-wave cavity, filled with a Kerr medium, with one perfectly reflecting mirror. As in the ring cavity, instability occurs with the driving field tuned between cavity resonances.

Ikeda's instability has generally been regarded as a dispersive phenomenon. However, my recent work [1] shows that, as with optical bistability itself, within a unified formulation using a two-level medium both absorptive and dispersive instabilities exist. Here I will treat the simplest case of a purely absorptive medium. In section 2 I develop the stability analysis for a standing-wave cavity containing a two-level homogeneously broadened absorber, arbitrarily positioned, and of arbitrary length. The analysis is carried out for the mean-field limit without the truncation of the Bloch hierarchy used in ref. [4]. In section 3 I discuss results for the adiabatic limit where the medium response time is much shorter than the cavity round-trip time. This is a convenient mathematical limit since general conclusions can be drawn from analytical expressions. Although, in this limit, positive branch instabilities in absorptive bistability cannot occur, multimode instability throughout the negative slope branch identifies their possibility outside the adiabatic limit [1]. Here the instability of Casagrande et al. [4] needs to be

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FG4 Intracavity optical bistability in the presence of optically induced absorption

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Optical bistability in the presence of an optically induced absorption is observed when a thin 13-μm CdS sample is inserted between two mirrors of 90% reflectivity. In our experiment, a modulated cw laser of ~15-mW peak power is tuned on each side of the free exciton resonance in CdS, and the transmission characteristics are studied. A theoretical calculation including both absorptive and dispersive effects is presented and good agreement is obtained. The absorption coefficient and the index of refraction of the material are connected by Kramers-Kronig relations. Propagation effects are taken into account. Sufficiently long pulses (40 ms) are considered to allow the thermally induced absorption to occur. For detunings above the free exciton resonance, we predict that counterclockwise hysteresis loops will first be observed as the intensity rises; for detunings below the free exciton resonance, only clockwise hysteresis loops are predicted. These predictions were experimentally verified. Power levels of the order of 5–10 mW were sufficient to observe bistability. Cavity effects were also observed. **(12 min)**

FG5 Experimental test of the single-Gaussian mode theory of optical bistability

L. A. OROZCO, A. T. ROSENBERGER, and H. J. KIMBLE, U. Texas at Austin, Physics Department, Austin, TX 78712.

In nonlinear optical interactions, one expects the transverse spatial intensity dependence to be modified. Nonlinear interactions within an optical resonator, in the absence of a mode-selective element such as an aperture, will thus in general couple the energy into various transverse modes of the resonator. One attractive analytical model incorporating transverse effects into optical bistability assumes, in spite of the observations above, that the transverse intensity profile remains a single-Gaussian (TEM_{00}) mode. This model seems to be supported by experimental results for bistability with two-level sodium atoms in a confocal ring resonator. To test the generality of the single-Gaussian mode model, we have performed bistability experiments with two-level atoms in standing-wave resonators. Two resonators, differing only in length, were used. The confocal resonator is transverse-mode-degenerate and will support a combination of modes, as would the ring; the slightly longer resonator will allow only a single-transverse mode to be excited. Preliminary results indicate that there may be a difference in the two cases; although the behavior near the critical onset of bistability seems to be the same in the two cases, there seems to be a difference in the evolution of hysteresis with increasing cooperativity parameter. Further results, including a quantitative comparison with the single-Gaussian mode theory for standing-wave resonators, are presented.

(12 min)

FG6 Theory of parametric instability in stimulated Raman scattering

J. T. LIN, JAYCOR, 205 S. Whiting St., Alexandria, VA 22304.

Parametric instability induced by the medium saturation in stimulated Raman scattering is investigated theoretically via a generalized detuning

$$\Delta = (\omega_p - \omega_s - \omega_o) + \phi_{ps} - (\varepsilon^*/\omega_o)|Q|^2, \qquad (1)$$

where $\omega_{p,s,o}$ are the frequency of the pump, Stokes, and phonon, $\dot{\phi}_{ps}$ is the phase modulation, and the last term is the nonlinear correction of the highly excited anharmonic molecule with an amplitude Q. The steady-state gain and hence the Stokes efficiency, in the presence of the anharmonic term, are governed by a cubic equation of $|Q|^2$ which is to be solved from the Bloch and Maxwell equations including the pump depletion. Numerical results are shown for the transition diagram of Stokes efficiency vs input pump intensity at various system conditions at the steady-state. For the transient regime, with pulse duration shorter than the dephasing time, we analyze the temporal profiles of the Stokes and the depleted pump based on a transformed equation which combines Bloch and Maxwell equation as follows:

$$\frac{d^2Y}{dW^2} + \frac{1}{W}\left(1 + \frac{2\Gamma T}{I_0}\right)\frac{dY}{dW} = \left(\frac{\Gamma G_s}{2I_0}\right)\sin Y - \left(\frac{2\Delta}{I_0}\right)U, \qquad (2)$$

where T is a nonlinear time, G_s is the steady-state gain, I_0 is the input pump intensity, and U is the real part of the phonon amplituda Q. For an input pulse intensity given by $\mathrm{sech}^2(t/t_p)$, Eq. (2) is solved numerically to study the transient pulse profiles in the presence of pump depletion and medium saturation. **(12 min)**

FG7 Quantum theory of optical bistability with a spatially varying field mode in the good cavity limit

XIAO MIN and H. J. KIMBLE, U. Texas at Austin, Physics Department, Austin, TX 78712; H. J. CARMICHAEL, U. Arkansas, Physics Department, Fayetteville, AR 72701.

A straightforward but general extension of the quantum theory of optical bistability is made to include spatial variations of the field mode in the good cavity limit. The analysis proceeds by dividing the field mode into small sections which are each microscopically large in terms of the atomic number to allow truncation of the generalized Fokker-Planck equation but which are macroscopically small to justify the assumption of constant field amplitude. In a linearized approximation, analytic expressions are obtained for the ratio of incoherent to coherent intensity and for the intensity correlation function of the transmitted field for the two particular examples of a Gaussian-mode field in a ring cavity and a plane-wave field in a standing-wave cavity. In the weak field limit the results of the plane-wave ring cavity are recovered independent of the form of the spatial dependence of the cavity mode. However, more generally a nonuniform distribution tends to suppress certain quantum features such as photon antibunching. **(12 min)**

Friday — **MORNING**

18 October 1985 — **FH**

MILITARY

9:45 AM **Holography: 3**

Sing H. Lee, Presider

FH1 Two-dimensional scanner using pseudo-plane holograms

B. H. ZHUANG, CHARLES S. IH and L. Q. XIANG, U. Delaware, Electrical Engineering Department, Newark, DE 19716.

Plane grating (plane hologram) scanners are easy to make and to operate and can produce very straight scan lines in a specific operation condition.[1] This limits their operation to 1-D scanning. For some applications, a 2-D scanning is more convenient and/or desirable. We describe a technique for improving the plane hologram scanner so that straight scan lines can be obtained over wider operating conditions. If the holograms are made with a spherical wave and a plane wave instead of two plane waves, the scan line straightness can be sufficiently improved for 2-D operation. By adjusting the divergence of the spherical wave at the hologram, compensations can be made over considerably large (vertical) deflection angles. This technique can be applied to scanners in which the construction and reconstruction wavelengths may be different. This is advantageous because many hologram recording media are only sensitive at shorter wavelengths but the reconstruction is more economical or convenient at a longer wavelength. Techniques for making the scanners are discussed. Computer simulations and experimental results for the performance of the 2-D scanner are shown.

(12 min)

1. C. J. Kramer, "Optical Scanner Using Plane Linear Diffraction Gratings on A Rotating Spinner," U.S. Patent 4,289,371 (15 Sept 1981).

FH2 Holographic tomography in determining the structure of 3-D object fields

F. T. S. YU, L. N. ZHENG, and T. W. LIN, Pennsylvania State U., Electrical Engineering Department, University Park, PA 16802.

An alternative method for determination of 3-D object fields is presented. The holographic multiplexing technique is employed in this method to record the multidirectional projections of the object field. The multiplexed hologram thus generated can be used to reconstruct the different directional projections of the studied field sequentially in the optical system. In addition, it provides series sets of projection data by means of an image sensing device for computer digital postprocessing. With the aid of a digital reconstructed image, the structure of the studied object field can be well determined. Illustrations of the recording and reconstructing systems as well as the algorithm adapted to reconstruct the field image digitally are given. Some preliminary results are also provided.

(12 min)

Journal of the Optical Society of America A – v 2, Issue 13, p100-p101, 1985

FT3 Phase-resolved resonance scattering and emission in iodine vapor

P. MITRA and A. Z. GENACK, Exxon Research & Engineering Co., Corporate Research Science Laboratories, Route 22 East, Clinton Township, Annandale, NJ 08801.

We address the distinction between resonance Raman scattering and fluorescence as the excitation source is continuously tuned through resonance. The experiments are performed in iodine vapor using a cw laser, whose intensity is electrooptically modulated in the 0.1–50-MHz frequency range. A lock-in amplifier is used to detect both the in-phase and out-of-phase response of the emission. The modulated signal is found to be the sum of two components. One component varies with angular modulation frequency ω, as $(1 + \omega^2\tau^2)^{-1/2}$, where τ is the lifetime of the excited state. The other is found to be flat within the range of modulation frequency used in the experiment. Since the results of the modulation experiment are the Fourier transform of the response to pulsed excitation, the first component corresponds to exponential decay with the fluorescence lifetime and the second corresponds to a prompt response. A density matrix calculation shows that the prompt response would be absent if the absorption were independent of laser frequency. It exists as a result of the finite inhomogeneous linewidth and the laser detuning from the center of the transition. The amplitude of the delayed and prompt components is a function of laser frequency. This gives a full description of the dynamical response to the emission to linear excitation as the laser is tuned continuously through resonance. The response of the sample to nonlinear excitation is also discussed. **(12 min)**

FT4 Q-branch spectral line shapes of D_2 in foreign gases

W. S. HURST and G. J. ROSASCO, U.S. National Bureau of Standards, Temperature and Pressure Division, Gaithersburg, MD 20899.

The collisional Dicke narrowing and broadening of spectral lines with foreign gas perturbers has been studied by measuring the third-order nonlinear susceptibility of isolated Q-branch (0–1) D_2 lines. Perturber gas mass has been varied by employing dilute (5%) binary mixtures of D_2 in H_2, He, N_2, or Ar, and densities at 296 K were varied from 0.25 to 5.7 amagat. We have used these data to test the validity of soft and hard collision models for describing the spectral line shape and the linewidth density dependence. At high densities, the data are well described by Lorentzian line shapes and a Dicke diffusion expression for the density dependence of the lindwidth. At lower densities, the line shape functions were fit to the data while fixing both the pressure broadening parameters to their high density values and the narrowing parameters to the values predicted from diffusion theory. For the light perturbing gases (H_2, He), only the soft collision model fit the line shape and gave agreement with the observed linewidth. For the heavy perturber gases (N_2,Ar), neither model described the line shape or the linewidth dependence, the disagreement being greater for Ar. **(12 min)**

FT5 Photodissociation cross section of Pb_2

GABRIEL G. LOMBARDI, Northrop Research & Technology, Quantum Electronics Department, Palos Verdes Peninsula, CA 90274.

A novel technique was used to measure the photodissociation cross section of Pb_2. This technique has the important advantage that the Pb_2 density need not be known. An excimer laser (XeCl) operating at 308 nm photodissociated the Pb_2, while a single-mode argon-ion laser probed the ground state dimer absorption at 496.5 nm. The Pb_2 was produced in a heat pipe at $\sim$1300°C. A similar technique was described by Kwong *et al.*[1] and applied to atomic transition probability measurements; however, their approach requires a measurement of the ground state atomic density. The technique reported in this paper makes use of the fact that the laser-induced change in the ground state Pb_2 density depends only on the fluence of the dissociating laser and the photodissociation cross section. Consequently, from a measurement of the relative change in the density as a function of fluence, the cross section may be deduced. The above analysis is valid in the approximation that the molecular vapor is optically thin at the frequency of the dissociating laser. The widespread availability of excimer lasers with emission into the vacuum ultraviolet should make this technique readily applicable to the measurement of absolute photoionization and photodissociation cross sections. **(12 min)**

1. H. S. Kwong, P. L. Smith, and W. H. Parkinson, Phys. Rev. A 25, 2629 (1982).

FT6 Molecular infrared coherence transfer kernels by scale transformation of photon echo decay data

JOHN E. THOMAS, JENG-MIN LIANG, LOUIS S. SPINELLI, RAMANCHANDRA R. DASARI, and MICHAEL S. FELD, MIT George R. Harrison Spectroscopy Laboratory, Cambridge, MA 02139.

Experiments exploiting similarities for coherence collisional evolution in velocity space and in M space are reported. Generally, there is a total outgoing rate from any state and incoming rates (kernels) from neighboring states. Collision-induced velocity changes, ΔV, are studied with lasers by measuring the Doppler frequency $\Delta V/\lambda$ (λ = optical wavelength). To obtain information about coherence M transfer, it is only necessary to associate a frequency with each ΔM. This idea is demonstrated by measuring two-pulse photon echo intensity I_e for $^{13}CH_3F$ in a small dc Stark field. Acoustooptic modulation of infrared cw laser radiation, polarized parallel to the Stark field, is used to excite a number of uncoupled $M \rightarrow M$ transitions on the $R(4,3)$ line. Each transition has a resonance frequency, $\nu o + M \cdot \nu s$ (νs is $M = 1$ Stark shift, Hz). A most remarkable result is obtained for large echo delays T with $\Delta VT/\lambda \gg 1$, all ΔV. The shape of the curve $\ln[I_e(o)/I_e(V)]$ vs Stark voltage V tends to that of the dominant coherence ΔM transfer kernel itself. Physically this arises because, for large T, the macroscopic coherence is preserved only for collisions where the frequency changing due to M transfer compensates the Doppler shift due to the ΔV, i.e., $\Delta V/\lambda + \Delta M \cdot \nu s = 0$. Generalizations are discussed. **(12 min)**

FT7 Los Alamos Fourier transform spectrometer

BYRON A. PALMER, Los Alamos National Laboratory, Chemistry Division, Los Alamos, NM 87545.

The Los Alamos Fourier transform spectrometer is a folded Michelson interferometer using cat's-eye reflectors for the moving mirrors. The maximum path difference is 5 m which gives a resolution of 0.001 cm^{-1}. The interferometer is capable of obtaining spectra from 200 to 20,000 nm with the appropriate optics and detectors. Acquisition time for one scan is 2–6 min depending on resolution and spectral region. The servo control of the interferometer uses a Zeeman split He–Ne laser in a phase lock loop operating at 1.5 MHz. The servo is capable of position accuracies of 0.3 nm and can acquire an interferogram with any reasonable free spectral range that is desired. The A/D converter is an effective 22-bit floating point converter connected to an array processor. The A/D converter control has self-calibration to ensure linearity over the full range. The array processor corrects for any errors in the A/D converter as determined by the calibration and then digitally filters the data to remove noise outside the bandpass of interest. The servo control and the A/D converter are operated by separate 68,000 microprocessors. **(12 min)**

FT8 Paper withdrawn

Friday **AFTERNOON**

18 October 1985

GEORGETOWN EAST

1:30 PM Optical Bistability: 2

Hyatt M. Gibbs, Presider

FU1 Photon antibunching in the single-atom limit of absorptive bistability

H. J. CARMICHAEL, U. Arkansas, Dept. of Physics, Fayetteville, AR 72701.

Photon antibunching has been predicted at weak intensities along the lower branch in optical bistability. Existing theories assume large atomic numbers to justify their linearized analysis. From such a theory photon antibunching in absorptive bistability is given for weak intensities by

$$g^{(2)}(0) - 1 = -\frac{4C}{N}\frac{\mu}{\mu + 1/2}\frac{2C}{1 + 2C}, \quad (1)$$

where C is the bistability parameter, μ is the ratio of atomic and cavity decay times, and N is the number of atoms. Clearly this result becomes invalid as the system size is reduced to the limit of a single atom; then $g^{(2)}(0)$ is negative for moderate values of C. I calculate $g^{(2)}(0)$ for the transmitted [illegible] from an optical cavity containing a single atom.

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Self-oscillation in a detuned cavity

By H. J. Carmichael

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A simple symmetry relates optical bistability in a ring cavity tuned near resonance and multimode instability and self-oscillation in a cavity excited midway between resonances.

It has been predicted that the continuous wave (c.w.) output of a bistable cavity may become unstable to multimode self-oscillation. Such multimode instabilities were first discussed by Bonifacio & Lugiato (1978) for absorptive bistability in a ring cavity. Their analysis was later extended by Lugiato (1980) to dispersive bistability. Recently, much attention has been given to this subject following Ikeda's identification of an instability leading to period-doubling and chaos in dispersive bistability (Ikeda 1979; Ikeda *et al.* 1980).

A detailed study of the work done by Ikeda led myself and co-workers to discover a second multimode instability for a saturable absorber in a ring cavity (Carmichael *et al.* 1982). A telling distinction exists between this and the instability studied by Bonifacio & Lugiato. They studied absorptive bistability in a resonant cavity. The instability that we have discovered occurs with the injected laser and resonant absorber tuned midway between cavity resonances. In a high-finesse cavity the Ikeda instability behaves similarly (Firth 1981; Carmichael *et al.* 1982; Bar-Joseph & Silberberg 1983). These observations provide the clue to the central result of this paper; the stability analysis for a nonlinear ring cavity exhibits a symmetry that establishes a one-to-one correspondence between optical bistability in a cavity tuned near resonance and multimode self-oscillation in a cavity tuned between resonances. The theory of absorptive and dispersive bistability can then be transferred as a whole to the description of corresponding multimode instabilities in a cavity tuned between resonances.

For simplicity I consider the plane-wave theory of absorptive bistability for a two-level homogeneously broadened medium in a ring cavity, and give detailed results only for the mean-field limit. My central conclusions are, however, quite general. They hold for dispersive bistability, for a gaussian-mode theory, and beyond the mean-field limit. On the other hand, they do not hold (at least not without qualification) in a standing-wave cavity, although it must be recognized that multimode instabilities have been predicted there also (Casagrande *et al.* 1980; Firth 1981).

The general stability analysis for a ring cavity containing a two-level homogeneously broadened absorber gives the following characteristic equation for eigenvalues λ governing the linearized dynamics (Carmichael 1983):

$$1 + R^2 e^{-2\lambda\tau}\left[\frac{E(L)}{E(0)}\right]^{2/(1+\lambda T_2)}\left[\frac{(1+\lambda T_1)(1+\lambda T_2)+E(0)^2}{(1+\lambda T_1)(1+\lambda T_2)+E(L)^2}\right]^{\frac{1}{2}(2+\lambda T_1)/(1+\lambda T_2)}$$
$$- R e^{-\lambda\tau}\left[\frac{E(L)}{E(0)}\right]^{1/(1+\lambda T_2)}\left\{1+\left[\frac{(1+\lambda T_1)(1+\lambda T_2)+E(0)^2}{(1+\lambda T_1)(1+\lambda T_2)+E(L)^2}\right]^{\frac{1}{2}(2+\lambda T_1)/(1+\lambda T_2)}\right\}\cos\theta = 0. \quad (1)$$

Optical Bistability and Multimode Instabilities

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A simple relationship transforms the theory of optical bistability into a theory of multimode instability and self-oscillation in a ring cavity excited between cavity resonances.

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It is now understood that optical bistability, as originally conceived, represents only the simplest amongst a variety of instabilities which can occur in an optical cavity containing a passive nonlinear medium. In particular, multimode instabilities, leading to various forms of self-oscillation, are possible. Bonifacio and Lugiato[1] first recognized this possibility on the basis of a multimode stability analysis for the mean-field theory of absorptive bistability in a ring cavity. Lugiato[2] extended this analysis to dispersive bistability. Ikeda[3] added the stimulus which focused attention on these instabilities when he found period doubling and chaos in a model for dispersive bistability in a ring cavity, treating cavity dynamics as a nonlinear mapping of the complex field amplitude on successive round trips.

Several authors have noted[4-6] that, in a high-finesse cavity, Ikeda's instability most readily occurs with the injected field tuned midway between cavity resonances. The fundamental period of oscillation, at twice the cavity round-trip time, is just the beat frequency between the injected field and the adjacent cavity modes. My intention in this Letter is to show that a very general symmetry underlies this observation. This symmetry identifies a whole class of multimode instabilities in a cavity tuned between resonances as counterparts to conventional optical bistability. For a short medium-response time, or a long cavity, where cavity dynamics are described by a nonlinear map, a one-to-one correspondence exists between bistable and oscillatory systems. The theory of both absorptive and dispersive bistability can then be transferred wholesale to the description of corresponding oscillatory instabilities. Moreover, the ensuing oscillations take place between states on the S-shaped bistability curves.

I consider the plane-wave theory of optical bistability in a ring cavity containing a homogeneously broadened two-level medium. The cavity is excited by a field with frequency ω_L propagating in the positive z direction. The medium has a resonant frequency ω_a, and fills the cavity from $z=0$ to $z=L$. Propagation through the medium is described by the Maxwell-Bloch equations

$$\frac{\partial E}{\partial z}+c^{-1}\frac{\partial E}{\partial t}=i\frac{\omega_L\mu}{2c\epsilon_0}P,$$
$$\partial P/\partial t=-i(\mu/\hbar)ED-T_2^{-1}(1+i\Delta)P, \qquad (1)$$
$$\partial D/\partial t=-2i(\mu/\hbar)(E^*P-EP^*)-T_1^{-1}(D+N),$$

and the cavity imposes the boundary condition

$$E(0,t)=E_i+Re^{-i\theta}E(L,t-\tau+L/c). \qquad (2)$$

Here E_i is the incident field amplitude after transmission at the input mirror, E is the cavity field amplitude, μP is the polarization amplitude, and D is the inversion; N is the atomic density, μ is the atomic dipole moment, T_1 and T_2 are atomic relaxation times, and $\Delta=T_2(\omega_a-\omega_L)$ is the atomic detuning; τ is the cavity round-trip time, R is the round-trip reflection loss, and $\theta=\tau(\omega_c-\omega_L)$ $[-\pi\leqslant\theta<\pi]$ is the phase detuning of the incident field from the nearest cavity resonance, with frequency ω_c.

To simplify what are otherwise rather long and tedious expressions, I will restrict my discussion to absorptive systems—$\Delta=0$—and the mean-field limit—$\alpha L<<1$, $(1-R)<<1$, with $C=\alpha L/4(1-R)$, where $\alpha=N\mu^2\omega_L T_2/\epsilon_0\hbar c$ is the resonant absorption coefficient. Neither of these restrictions is necessary, however. The results which follow can readily be generalized.

For the mean-field limit the steady-state solution to Eqs. (1) is found with $E=E_{ss}+zO(1-R)$, where E_{ss} is independent of z. With the incident field tuned to a cavity resonance ($\theta=0$) Eq. (2) requires that the cavity field amplitude satisfy

$$X[1+2C/(1+X^2)]=Y, \qquad (3)$$

where $X=(2\mu/\hbar)(T_1T_2)^{1/2}E_{ss}$ and $Y=(1-R)^{-1}\times(2\mu/\hbar)(T_1T_2)^{1/2}E_i$. With the incident field tuned midway between cavity resonances ($\theta=-\pi$) the second term in Eq. (2) changes sign, and

$$X=Y', \qquad (4)$$

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Analytic Solutions for Two-Photon Absorption in Counterpropagating Beams in the Cubic Approximation

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Exact stationary-state solutions of the cubic (unsaturated) nonlinearity model of degenerate two-photon absorption by counterpropagating beams are found, and are used to describe power limiting and multiple optical bistability within a Fabry-Perot etalon.

0. Introduction

The behaviour of an electromagnetic wave travelling within a two-photon absorbing medium is significantly different from that associated with a single-photon absorbing medium. In particular an unsaturated two-photon absorber modelled by the two-photon Bloch equations in the paraxial and slowly-varying amplitude approximations, where the medium is detuned from the field frequency by an amount δ, is described [1, 2] by a stationary complex field $E(z)\exp(i\phi(z))$ such that the intensity $I(z)=|E(z)|^2$ and phase $\phi(z)$ satisfy

$$dI/dz=-\alpha(1+\delta^2)^{-1}\cdot I^2; \tag{1a}$$

$$d\phi/dz=\tfrac{1}{2}\alpha\delta(1+\delta^2)^{-1}\cdot I \tag{1b}$$

where α is the two-photon absorption coefficient. The solution of (1a) is the hyperbolic profile

$$I(z)=I(0)/\{1+I(0)\,\alpha z/(1+\delta^2)\}. \tag{2}$$

In terms of the field-amplitude dependence of the polarisation density, (1a) describes a cubic nonlinear effect. The plane-wave intensity profile (2) differs from the linear-absorption profile in being bounded above by $(1+\delta^2)/\alpha z$ at every point within the medium as $I(0)\to\infty$ (note that $\partial I(z)/\partial I(0)\to 0$ in the same limit). This feature has been shown to have an important effect upon transmission characteristics in the theory of two-photon optical bistability in a ring cavity [3], and also suggests an application for a two-photon device as a power limiter [4]. The feasibility of constructing a power limiter which utilises both nonlinear absorptive and refractive effects has arisen in consequence of certain recent cryogenic and room-temperature experiments with semiconductors [5].

In this paper we present analytic solutions for the stationary field distribution for counterpropagating waves in an unsaturated two-photon absorber, and use our results to formulate a theory of two-photon optical bistability within a folded Fabry-Perot (FP) cavity driven by a cw laser. We will treat two examples of cubic coupling between forwards and backwards wave amplitudes, corresponding to: (a) a homogeneously-broadened medium, where the absorption of both counterpropagating and copropagating photon pairs is included, and (b) a Doppler-broadened medium where the absorption of counterpropagating photon pairs dominates. In the former case transmission characteristics for a bistable FP cavity are power-limited with an upper bound as in a bistable ring cavity, while in the latter case this feature is absent. However, in the latter case there is a damping of the nonlinear effects associated with higher-order cavity resonances [20] due to the increase in round-trip absorption with increasing cavity intensity. The various physical mechanisms for cubic nonlinearities other than two-photon absorption will

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several interesting nonlinear optical devices. For a thin medium, we have calculated the far-field transmitted intensity distribution as a function of the laser beam waist, radius of curvature, nonlinear refraction index, geometrical configuration, and the feedback from an external mirror.[1] We have determined the ranges of values of these parameters for which the system will act as a differential gain device, a power limiter, and a bistable switch. Without the feedback, the system will allow transistor action using opto-optical modulation. Experimentally, we have demonstrated these results using a thin film of nematic liquid-crystal film in conjunction with a low-power cw laser. A separate study using nanosecond laser pulses demonstrates that the nematic could function as microsecond devices with moderate driving laser power (kilowatts). Experimentally observed switching curves and other nonlinear output/input characteristics are in good agreement with theory. (12 min.)

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WT2. Optical Bistability and Multimode Instabilities in a Standing-Wave Cavity. H. J. CARMICHAEL, *Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701.*—It has recently been shown[1] that the stability analysis for a ring cavity exhibits a symmetry that transforms optical bistability in a resonant cavity into a multimode oscillatory instability in a cavity tuned midway between resonances. This correspondence unifies a number of seemingly different instabilities. I show that a similar relationship organizes instabilities in a standing-wave cavity. I analyze the stability of a standing-wave cavity containing a two-level homogeneous broadened medium. In the limit of a medium response much faster than the cavity round-trip time and a medium length much shorter than the cavity length a great variety of multimode instabilities are possible depending on the position of the medium. If $l/2$ is the distance from the input mirror to the medium and $L/2$ is the cavity length, multimode instability exists over the range of intracavity field amplitude X defined by the turning points in optical bistability whenever l/L is a rational number. When l/L is expressed as the ratio of two odd integers corresponding instabilities exist in a resonant cavity—leading to bistability—and in a cavity tuned between resonances—leading to self-oscillation. This correspondence results from a symmetry analogous to that in a ring cavity.[1] (12 min.)

[1] H. J. Carmichael, Phys. Rev. Lett. 52, 1292 (1984).

WT3. What Is the Best Measure of Cavity Loss in Optical Bistability?* A. T. ROSENBERGER, L. A. OROZCO, AND H. J. KIMBLE, *Department of Physics, The University of Texas at Austin, Austin, Texas 78712.*—In order to calculate the cooperativity parameter C, one needs to know the cavity loss; it can be measured in several ways. Both steady-state measurements, such as the peak transmission and finesse of the cavity, and transient measurements, such as the cavity's response to an input switched on or off, may be made. These measurements may give various values of the cavity loss, differing by a factor of 2 or more. The reasons for this are discussed, with the confocal cavity treated in detail (these results also apply, e.g., to the scanning spectrum analyzer). Experimental results for confocal standing-wave and ring cavities are presented as examples of the above and are interpreted in terms of a simple model taking mirror aberrations and surface figure into account. It will be seen that the cavity has one time constant that dominates the decay and another that governs the filling of the cavity. In general, the relaxation response to a perturbation from steady state depends on the size of the perturbation. This is interpreted in the context of the state equation of optical bistability, switching times, and self-pulsing, and an answer to the title question is given, at least for the steady-state case. (12 min.)

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WT4. Observation of the Single-Mode Instability in Optical Bistability.* L. A. OROZCO, A. T. ROSENBERGER, AND H. J. KIMBLE, *Department of Physics, The University of Texas at Austin, Austin, Texas 78712.*—Self-pulsing due to the single-mode instability has been observed in optical bistability. Our bistable system consists of a confocal ring interferometer, excited by a cw dye laser, through which ten highly collimated beams of optically pumped sodium atoms pass. This provides, to our knowledge, the closest experimental realization of the canonical theoretical model of optical bistability with two-level atoms in a ring cavity. For values of the cooperativity parameter C greater than 90, and with small atomic and large cavity detunings of opposite sign (e.g., $\Delta = 1$, $\theta = -10$), self-pulsing is observed on the upper branch. It occurs within the bistable region and above it and also appears when the detunings are large enough that there is no bistability. Observation of the cavity transmission in time or on an rf spectrum analyzer, while remaining at a fixed point on the output versus input curve, shows a nearly sinusoidal oscillation at a frequency ranging from 20 to 60 MHz. Preliminary results for the range of existence of the instability, the frequency and modulation depth of self-pulsing, and the development of the instability from anomalous behavior (kinks) on the upper branch are described and compared with the predictions of theory. (12 min.)

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WT5. Grating and Prism Input Couplers as Bistable Integrated Optical Elements. W. LUKOSZ, *Swiss Federal Institute of Technology, Professur für Optik, ETH, 8093 Zürich, Switzerland.*—We propose a novel class of intrinsic dispersive and absorptive bistable elements, viz., grating or prism couplers on nonlinear planar waveguides. For sufficiently high input powers P, the power P' of the guided wave becomes a bivalued function of P. The incoupling efficiency P'/P has a resonance peak as a function of the effective refractive index N of the guided mode. The index N, in turn, depends on the incoupled power P' through (i) a nonlinearity of the refractive index of waveguide or cover, caused by the Kerr effect, saturation of absorption, or the thermo-optic effect or (ii) light- or temperature-induced desorption of molecules from the waveguide surface.[1] Initially detuned from resonance by appropriate choice of the angle of incidence, the system is driven into resonance with increasing power P. This is the origin of the mainly dispersive bistability. Absorptive bistability results from saturating the strong absorption of a thin layer covering the waveguide in the coupling area by the guided power P'. The input couplers function as hybrid bistable elements when a feedback signal derived from the guided power P' is used to generate a change in N through an electro-optic, thermo-optic, or desorption effect. (12 min.)

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WT6. New Radiation Pressure-Induced Mechano-Optical Bistability. W. LUKOSZ, *Swiss Federal Institute of Technology, Professur für Optik, ETH, 8093 Zürich, Switzerland.*—A Fabry-Perot resonator with one mirror moving under the influence of the radiation pressure has been shown to be bistable.[1] We propose and analyze a new kind of mechano-optical bistability: A planar waveguide on a light-weight substrate is suspended to swing as a torsional pendulum. A laser beam is coupled into the waveguide by a grating or prism input coupler at one end and is outcoupled at the other end of the waveguide. The radiation pressure exerts a torque on the pendulum that is proportional to the incoupled power. The incoupling efficiency has a sharp angular resonance. Therefore, the system is a sensitive angle-position sensor. The torque turns the pendulum and, thus, changes the angle of incidence. When, initially, the pendulum is detuned from resonance, sufficiently high laser powers drive it into resonance, and bistability occurs. For still higher laser powers the radiation pressure stabilizes the angular position of the pendulum, greatly suppressing its oscillations under the influence of vibrational and thermal noise. Potential applications of this stabilization effect are discussed. (12 min.)

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