Conditional homodyne detection of light with squeezed quadrature fluctuations

Justin Vines, Reeta Vyas, and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701, USA
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We discuss the detection of field quadrature fluctuations in conditional homodyne detection experiments and possible sources of error in such an experiment. We also present modifications to these experiments to help eliminate such errors and extend their range of applicability.

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I. INTRODUCTION

Fluctuations of light provide a window on the underlying quantum dynamical evolution of a light-emitting source. Emission of a photon by a light source signals quantum fluctuations in progress, and a measurement that is conditioned on a photodetection allows us to study the time evolution of the fluctuations [1]. For example, measurement of light intensity conditioned on a photodetection, also known as two-time intensity correlation, reveals information regarding bunching and antibunching that is not available in unconditional intensity measurements [2–6].

The conditional measurement of quadrature fluctuations (CMQF) proposed by Carmichael *et al.* [7] reveals in a novel way the nonclassical nature of light from a cavity containing two-level atoms; this has been experimentally observed by Foster *et al.* [8]. Nonclassical effects in conditional intensity and squeezing in light from a degenerate parametric oscillator have been studied [6]. The conventional methods of detecting quadrature squeezing involve unconditional measurements that are degraded by detection inefficiencies and do not explore the time evolution of quadrature fluctuations [1,2,9,10]. The CMQF, on the other hand, is essentially independent of detection efficiency and provides a sensitive probe of the fluctuations' development over time. It has been shown that the conditional measurement can reveal remarkable nonclassical behavior of not only squeezed but also of unsqueezed quadrature fluctuations [6–8].

In Sec. II, we briefly summarize the theoretical concepts underlying a CMQF experiment [6,7]. We then consider intracavity second harmonic generation (ISHG) [11–13] and present a theoretical analysis of the nonclassical features of ISHG quadrature fluctuations.

The CMQF technique achieves a measurement of the quadrature fluctuations of a given source field by cross-correlating a photon count with a balanced homodyne detection. This technique requires the use of auxiliary coherent oscillators (coherent laser sources), and the amplitudes or intensities of the coherent oscillator fields must be set to values that depend on the properties of the source field. It can be shown that the accuracy of the measurement's final results is extremely sensitive to the precision of these adjustments. In some cases, a very small error in these adjustments may give rise to incorrect conclusions about the state of the source field's quadrature fluctuations. We demonstrate this in Sec. III by developing a theoretical model of such an error in the CMQF measurement and exploring its effects on conclusions that might be drawn about the state of the ISHG field.

Weak fields with approximately Gaussian fluctuations are ideal candidates for accurate quadrature fluctuation measurements using the CMQF method, provided that coherent field adjustments can be made sufficiently precise. However, the technique is limited in its applicability to a source field with nonzero third-order moments [14,15], which may obscure the results of the measurement, even with perfect control of coherent light parameters. In Sec. IV, we propose an extension of the CMQF method that can achieve a measurement of the quadrature fluctuations of a completely generic source field while eliminating the effects of third-order fluctuation moments. Additionally, this extended CMQF measurement avoids the need for precise adjustments of coherent laser fields, thus perhaps averting the kinds of error discussed in Sec. III. In Sec. V, we summarize our findings.

II. CONDITIONAL MEASUREMENT OF QUADRATURE FLUCTUATIONS FOR ISHG

The quadrature variables for an optical field with annihilation and creation operators $\hat{a}_s$ and $\hat{a}_s^\dagger$ are defined by

$$\hat{X}_\phi = \tfrac{1}{2}(e^{-i\phi}\hat{a}_s + e^{i\phi}\hat{a}_s^\dagger), \tag{1}$$

$$\hat{Y}_\phi = \frac{1}{2i}(e^{-i\phi}\hat{a}_s - e^{i\phi}\hat{a}_s^\dagger) = \hat{X}_{\phi+\pi/2}, \tag{2}$$

where ϕ is an arbitrary phase [2]. It follows from this quadrature definition that the variances $\langle:(\Delta\hat{X}_\phi)^2:\rangle$ and $\langle:(\Delta\hat{Y}_\phi)^2:\rangle$ are related to the intensity of the field fluctuations $\langle\Delta\hat{a}_s^\dagger\Delta\hat{a}_s\rangle$ by

$$\langle\Delta\hat{a}_s^\dagger\Delta\hat{a}_s\rangle = \langle:(\Delta\hat{X}_\phi)^2:\rangle + \langle:(\Delta\hat{Y}_\phi)^2:\rangle, \tag{3}$$

where colons denote time and normal ordering of the operators enclosed by them. For classical fields, both quadrature variances are always greater than or equal to zero, being equal to zero only in the classical coherent state. For quantum fields, however, the normally ordered variance of a quadrature $\hat{X}_\phi$ can become negative as long as the normally ordered variance of $\hat{Y}_\phi$ increases in such a way that Eq. (3) is still satisfied. In such a case, the quadrature $\hat{X}_\phi$ is said to be squeezed and the field $\hat{a}$ is said to be in a squeezed state [2]. This fact and the fact that the fluctuation intensity is nonnegative lead to the inequality [6,7]

Nonlinear dynamics of a modulated bidirectional solid-state ring laser

Fuad Rawwagah and Surendra Singh

Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701

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We investigate the dynamical behavior of a class-B, bidirectional, solid-state ring laser with a square-wave modulated pump. Our treatment includes the coupling of oppositely directed traveling wave modes via backscattering in addition to their coupling via the gain medium. We find that depending on the pump ratio and the depth and frequency of modulation, the intensity waveforms of the two oppositely directed modes may exhibit periodic, quasi-periodic, and chaotic behavior. We also find that although the periodic waveforms of mode intensities are antisynchronized, chaotic waveforms may be synchronized or unsynchronized. A detailed map of different operating regimes as functions of frequency and depth of modulation is presented. Curves are presented to illustrate the behavior. © 2006 Optical Society of America

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1. INTRODUCTION

Interaction of atoms and field in a laser may produce a great variety of periodic or aperiodic intensity waveforms in the output of a laser. In particular, lasers with modulated pump or loss are capable of generating nonlinear oscillations as well as chaotic waveforms, depending on the operating parameters of the laser. Chaos in single-mode solid-state lasers with modulated loss or gain was investigated theoretically by Khanin and coworkers,[1–5] and period doubling and chaotic emission were observed experimentally by Klische and coworkers[6,7] in a standing-wave NdP_5O_{14} laser with a periodically modulated pump.

Nonlinear dynamics of solid-state ring lasers operating in a single mode in both directions have also been investigated, both experimentally and theoretically.[8–14] Larionstsev and coworkers have investigated relaxation oscillations and dynamical chaos in bidirectional monolithic Nd:YAG ring lasers in a series of papers.[8–10] Using a cosinusoidal pump modulation, they investigated the appearance of dynamical chaos and attributed its appearance to parametric interactions between self-modulation and relaxation oscillations.

One major reason for the instability of a single-mode bidirectional ring laser with a homogeneously broadened gain medium is the coupling of oppositely traveling waves due to backward scattering of one wave (backscattering) in the direction of the other off the optical elements inside the cavity.[11] This coupling leads to a spatial modulation of gain via a population-inversion grating. If this grating is a small modulation of the overall population inversion, it can be approximated by a sinusoidal function. Zeghlache and Mandel used this approximation and adiabatic elimination of the polarization to derive and solve the equations of motion for a CO_2 ring laser.[12] They found that although the bidirectional steady-state operation is unstable, the unidirectional operation can be stable or unstable, depending on the operating parameters of the gain medium and the resonator.

In this work we investigate the nonlinear dynamics of a single-mode bidirectional solid-state ring laser (SSRL) under the influence of square-wave modulation of laser gain. This is different from previous investigations that considered sinusoidal modulation of gain. It is known from earlier studies[15] that the shape of modulation can profoundly affect nonlinear behavior. Indeed, with square-wave modulation, we find a phase-space portrait in the parameter space spanned by the frequency and depth of modulation that is different from that with sinusoidal modulation.

The paper is organized as follows. In Section 2 we present the equations of motion for the slowly varying field amplitudes and population inversion for the SSRL based on semiclassical laser theory. We consider the solution of these equations for a Nd:YAG ring laser with gain modulation in Section 3 and numerically explore the dynamical behavior they predict over a wide range of modulation frequencies and depths. Its behavior at different levels of excitations is characterized in terms of Lyapunov exponents and spectral densities for the counterpropagating waves. The results are summarized in Section 4.

2. EQUATIONS OF MOTION

Consider an optically pumped SSRL that supports one longitudinal mode in each direction. These modes interact with a collection of two-level atoms. To describe the dynamical evolution of this system, we use a semiclassical approach in which the field is treated classically using Maxwell's equations, and the atoms of the active medium are treated quantum mechanically using density matrix-equations of motion. Assuming that the mode frequencies ω_1 and ω_2 are close to the atomic transition frequency $\omega_0(\omega_1 \approx \omega_0 \approx \omega_2 \equiv \omega)$, we can take both modes to have nearly the same propagation constant. We can then write the electric field as the sum of two oppositely directed traveling waves:

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On the bichromatic excitation of a two-level atom with squeezed light

Amitabh Joshi[a], Reeta Vyas, Surendra Singh, and Min Xiao

Department of Physics, University of Arkansas, Fayetteville AR 72701, USA

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Abstract. Analytical results for the dynamical evolution of a single two-level atom coupled to an ordinary heat bath and bichromatically excited with two finite bandwidth squeezed fields are presented by solving the Heisenberg equations of motion. Photon statistics of the system in terms of the second-order intensity-intensity correlation function are also discussed. Transient fluorescent intensity as well as intensity correlation function exhibit oscillatory phenomenon even in the weak field limit. The latter also shows enhanced delayed bunching effect. All these effects are sensitive to the bandwidth of squeezed light.

PACS. 42.50.Ct Quantum description of interaction of light and matter; related experiments – 42.50.Dv Nonclassical states of the electromagnetic field, including entangled photon states; quantum state engineering and measurements

1 Introduction

Studies related to the dynamical evolution, the photon statistics, spectral and radiative properties of one and many two-level atoms embedded in a broadband squeezed bath have been the topics of keen interest in quantum optics. Gardiner [1] studied the interaction of a two-level atom with a broadband squeezed bath and predicted unequal polarization quadrature-decay rates. Carmichael, Lane and Walls [2] were able to discover a significant phenomenon of sub-natural linewidth in the fluorescence spectrum of a driven two-level atom, in the presence of squeezed light. Atomic absorption spectrum was discussed by Ritsch and Zoller [3] in the presence of colored squeezed vacuum. Since then many interesting results in atom-squeezed field interaction have been reported which include both two and three-level atoms interacting with broad bandwidth or narrow bandwidth squeezed baths [4, 5]. In a recent study the interaction of a two-level atom with the squeezed vacuum of bandwidth smaller than the natural atomic linewidth was considered and the hole burning and the three-peaked structure in spectra of fluorescence and transmitted field were predicted [6]. These results essentially show that squeezed fields having pairwise correlations and anisotropic noise distribution can give rise to interesting phenomena including novel features in spectral properties of atoms, formation of pure states and photon statistics. A more realistic model of finite bandwidth squeezed light interacting with a single two-level atom has been studied by Vyas and Singh [7] and Lyublinskaya and Vyas [8] where the source of the squeezed light employed was a degenerated parametric oscillator (DPO) operating below threshold and a homodyned DPO [9]. In another work, a two-level atom inside an optical parametric oscillator has been considered and hole and dips in the fluorescence and transmitted light has been observed [10]. The interest in other sources of squeezed light as well as its applications in a wide variety of areas has continued unabated [11–13].

The interaction of a single two-level atom with bichromatic driving field has also been studied extensively both theoretically and experimentally [14]. These studies were motivated by the observations that the bichromatic nature of the driving field can lead to a number of novel features which are different from the monochromatic case. For example, the fluorescence intensity exhibits resonances at subharmonics of the Rabi frequency and different spectral characteristics when compared with the usual Mollow triplet. Recently, some new calculations for resonance fluorescence and absorption spectra of a two-level atom driven by bichromatic field have been reported [15]. Also, reported are the effects of broadband squeezed reservoir on the second order intensity correlation function and squeezing in the resonance fluorescence for a bichromatically driven two-level atom [16]. Coherent population trapping and Sisyphus cooling under bichromatic illumination have also been studied [17]. In another recent work the electromagnetically induced transparency (which normally occurs in three-level atoms) has been demonstrated in a two-level atom excited by a bichromatic field (one strong and one weak field) and possibility of squeezed-light generation has also been discussed [18].

In this work, we study the interaction of a single two-level atom with a bichromatic electromagnetic field that is

[a] e-mail: ajoshi@uark.edu

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Conditional measurements as probes of quantum dynamics

Shabnam Siddiqui, Daniel Erenso, Reeta Vyas, and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701, USA
(Received 5 December 2002; published 17 June 2003)

We discuss conditional measurements as probes of quantum dynamics and show that they provide different ways to characterize quantum fluctuations. We illustrate this by considering the light from a subthreshold degenerate parametric oscillator. Analytic results and curves are presented to illustrate the behavior.

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I. INTRODUCTION

Fluctuations are an integral feature of quantum dynamical evolution. For dissipative quantum optical sources, these fluctuations are reflected in the photoemissions from these sources. In this paper, we will use the terms photoemissions and photodetections interchangeably because photodetection is simply a matter of casting a net to catch the photons emitted by the source. A photoemission signals a fluctuation in progress. Hence, a conditional measurement that commences when a photoemission has occurred catches a fluctuation in the act and, in fact, allows us to observe the time evolution of the fluctuation [1]. This sort of information is not available in unconditioned measurements. Thus, conditional measurements allow us to probe quantum dynamics at a deeper level. Perhaps the best known example of conditional measurement is the measurement of the second-order intensity correlation that measures fluctuations of light intensity following a conditioning photodetection [2]. This is the basis of the observation of photon bunching or antibunching. Recently, measurements of quadrature squeezing conditioned on a photodetection have also been proposed and reported in cavity quantum electrodynamics [3,4]. In this paper, we consider conditional measurements in the context of parametric oscillators and show that such measurements provide sensitive probes of quantum dynamics and the language of conditional measurements provides powerful conceptual tools for unraveling and understanding nonclassical features of quantum dynamics [5–8].

Optical parametric oscillators (OPOs) and amplifiers [9] based on down-conversion have played a central role in the studies of nonclassical photon correlations and various schemes for quantum communication and computation [10]. The fundamental process in optical parametric oscillators is conversion of a pump photon of frequency ω_p into a pair of photons (signal and idler) of lower frequencies ω_s and ω_i in a nonlinear medium inside an optical cavity. The process conserves energy and momentum. Conservation of energy requires the pump and down-converted frequencies to be related by $\omega_p = \omega_s + \omega_i$ and conservation of momentum, also known as phase matching, requires the use of certain anisotropic material media and specific states of polarization for the pump, signal, and idler photons. If the down-converted photons have the same frequency ($\omega_s = \omega_i \equiv \omega_d$), polarization, and direction of propagation, the process is called degenerate otherwise it is called nondegenerate. In the former case, we speak of a degenerate parametric oscillator (DPO) and in the latter of a nondegenerate parametric oscillator.

We begin by considering a degenerate parametric oscillator in Sec. II and discuss conditional measurements of intensity and amplitude. Conditional measurements of quadrature fluctuations are discussed in Sec. III. We restrict our considerations to its operation below the threshold of sustained oscillations. This allows us to obtain simple analytic expressions for various quantities of interest. The results are summarized in Sec. IV.

II. CONDITIONAL MEASUREMENTS OF A DPO

The field from the DPO is governed by the interaction Hamiltonian for phase-matched down-conversion inside an optical cavity driven by a classical injected signal [5,6]. The equation of motion for the density operator $\hat{\rho}_d$ of the DPO field is then

$$\dot{\hat{\rho}}_d = \frac{\kappa\varepsilon}{2}[\hat{a}_d^{\dagger 2} - \hat{a}_d^2, \hat{\rho}_d] + \gamma(2\hat{a}_d\hat{\rho}_d\hat{a}_d^\dagger - \hat{a}_d^\dagger\hat{a}_d\hat{\rho}_d - \hat{\rho}_d\hat{a}_d^\dagger\hat{a}_d), \tag{1}$$

where κ is the mode-coupling constant, ε is the dimensionless amplitude of the classical pump field, γ is the cavity linewidth, and $\hat{a}_d$ and $\hat{a}_d^\dagger$ are the annihilation and creation operators for the DPO. In writing the equation of motion for the density matrix, we have neglected pump depletion that is a reasonable approximation for low down-conversion efficiencies and subthreshold operation of the DPO considered here. The combination $\kappa\varepsilon$ has been chosen to be real by a suitable definition of the phases of $\hat{a}_d$ and $\hat{a}_d^\dagger$.

Using the positive-P representation for the density matrix, we can map the equation of motion for the annihilation and creation operators for the DPO field onto a set of stochastic equations for the c-number amplitudes α_d and α_{d*} corresponding, respectively, to the annihilation and creation operators $\hat{a}$ and $\hat{a}_d$. These equations read [5,6]

$$\dot{\alpha}_d = -\gamma\alpha_d + \kappa\varepsilon\,\alpha_{d*} + \sqrt{\kappa\varepsilon}\,\xi_1(t), \tag{2}$$

$$\dot{\alpha}_{d*} = -\gamma\alpha_{d*} + \kappa\varepsilon\,\alpha_d + \sqrt{\kappa\varepsilon}\,\xi_2(t), \tag{3}$$

where $\xi_1(t)$ and $\xi_2(t)$ are two statistically independent real Gaussian white-noise processes with zero means and unit intensity. Normally ordered averages of $\hat{a}_d^\dagger$ and $\hat{a}_d$ can then be calculated by using the mapping

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Quantum well in a microcavity with injected squeezed vacuum

Daniel Erenso, Reeta Vyas, and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701
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A quantum well with a single exciton mode in a microcavity driven by squeezed vacuum is studied in the low exciton density regime. By solving the quantum Langevin equations, we study the intensity, spectrum, and intensity correlation function for the fluorescent light. An expression for the Q function of the field inside the cavity is derived from the solutions of the quantum Langevin equations. Using the Q function, the intracavity photon number distribution and the quadrature fluctuations for both the cavity and fluorescent fields are studied. Several interesting and new effects due to squeezed vacuum are found.

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I. INTRODUCTION

With the development of semiconductor optical microcavities, there has been considerable interest in exciton-cavity coupled systems [1,2]. These systems have revealed some interesting phenomena that are similar to those observed in the interaction of a two-level atom with light [3–9]. The exciton-cavity system gives rise to the so-called polaritons, which are the normal modes of a coupled exciton-photon system. The excitation spectrum of the composite exciton-cavity system is characterized by two well-resolved polariton resonances (or normal mode resonances) when $g>(\gamma_e,\gamma_c)$, where g is the dipole coupling between the exciton and the cavity mode, and γ_e and γ_c are the exciton and cavity mode damping rates, respectively. In this limit, an excitation of the cavity mode can lead to a coherent oscillatory energy exchange (or normal mode oscillation) between the exciton and the cavity due to the vacuum Rabi oscillations.

The vacuum Rabi oscillations in a coupled exciton-photon system in semiconductor microcavity lasers have been observed by Weisbuch *et al.* [2]. Following this observation, extensive experimental and theoretical studies have been carried out [10–17]. These studies have confirmed normal mode splitting and oscillatory emission from exciton microcavities. Theoretical investigations in the linear regime, where the excitons can be approximated as bosons, have been carried out by Pau *et al.* [15]. Wang *et al.* [16] investigated the effects of inhomogeneous broadening of excitons on normal mode oscillations in semiconductor microcavities using the coupled oscillator model. Their results show that inhomogeneous broadening can drastically alter the coherent oscillatory energy exchange process even in regimes where normal mode splitting remains nearly unchanged.

In this paper, we study the excitonic system in a microcavity where the cavity is driven by squeezed vacuum. An outline of the system is shown in Fig. 1. A semiconductor quantum well is embedded between two Bragg reflecting mirrors. One of these mirrors acts as an input port through which light in a squeezed vacuum state is injected into the cavity. We include dissipation of both the cavity and exciton modes. In Sec. II, we derive the quantum Langevin equations for the exciton and cavity modes. We solve these equation for the case in which the damping constants are equal. These results are used in Sec. III to study the effects of initial cavity photon number as well as squeezed-vacuum photon number on the intensity, spectrum, and the second-order intensity correlation of the fluorescent light. In Sec. IV, we obtain the Q-distribution function and use it to study the intracavity photon number distribution and squeezing of the cavity mode and the fluorescent light. We summarize the principal results of the paper in Sec. V.

II. QUANTUM LANGEVIN EQUATION

We consider a semiconductor quantum well (QW) in the linear excitation regime where the density of excitons is small so that exciton-exciton interaction can be ignored. The excitons can then be approximated as a dilute boson gas [18]. In this approximation, the microscopic Hamiltonian in the interaction picture describing the exciton-cavity system is given by [17,19]

$$\hat{H}_I=\hbar\Delta\omega\hat{b}^\dagger\hat{b}+i\hbar g(\hat{a}^\dagger\hat{b}-\hat{a}\hat{b}^\dagger)+\hat{a}\hat{\Gamma}_c^\dagger+\hat{a}^\dagger\hat{\Gamma}_c+\hat{b}\hat{\Gamma}_e^\dagger+\hat{b}^\dagger\hat{\Gamma}_e. \tag{1}$$

The Hamiltonian of Eq. (1) is written in the rotating-wave approximation and in the dipole approximation. Here $\hat{a}$ and $\hat{b}$ are the annihilation operators for the cavity and exciton modes, respectively, in a frame rotating at frequency ω_c, $\hat{\Gamma}_c$ ($\hat{\Gamma}_e$) is the reservoir operator responsible for cavity field (exciton) damping, g is the coupling constant characterizing the strength of interaction between the exciton and the cavity field, and detuning $\Delta\omega=(\omega_e-\omega_c)$, where ω_e and ω_c are the frequencies of the exciton and cavity modes, respectively. Normally, the exciton and cavity modes are coupled to a continuum of thermal reservoir modes. This leads to their

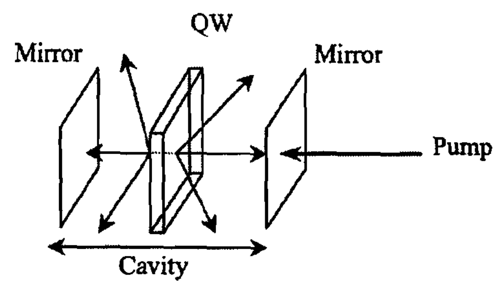

FIG. 1. An outline of the physical system.

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Higher-order sub-Poissonian photon statistics in terms of factorial moments

Daniel Erenso, Reeta Vyas, and Surendra Singh

Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701

Received September 10, 2001

We introduce the concept of higher-order super-Poissonian and sub-Poissonian statistics and show that higher-order sub-Poissonian statistics is a signature of a nonclassical field. Fields generated in intracavity second-harmonic generation and single-atom resonance fluorescence are shown to exhibit higher-order sub-Poissonian statistics. © 2002 Optical Society of America

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1. INTRODUCTION

Nonclassical properties of electromagnetic fields receive a great deal of attention, as these properties provide a testing ground for the predictions of quantum electrodynamics.[1,2] Sub-Poissonian Photon statistics based on second-order intensity correlations provide one way to characterize the nonclassical nature of a light beam.[3,4] Nonclassical effects as they relate to higher-order moments have also been discussed in the literature.[5–9] Agarwal and Tara discussed higher-order nonclassical effects for single-mode fields in terms of normally ordered moments.[5] Perina and co-workers studied nonclassical behavior in optical parametric processes, Raman and Brillouin scattering,[6] and nonlinear optical couplers[7] in terms of moments of integrated intensity. Lee considered single-mode fields and defined higher-order nonclassical effects in terms of factorial moments of the photon distribution by using majorization theory.[8] Vyas and Singh considered higher-order nonclassical effects in terms of factorial moments of the photocount distribution.[9] In this paper we introduce criteria for evaluating higher-order sub-Poissonian statistics in terms of factorial moments and show that higher-order sub-Poissonian statistics are indicative of nonclassical fields. The criteria that we introduce are independent of the efficiency of detection. We then show that the light from intracavity second-harmonic generation (ISHG) and light from single-atom resonance fluorescence exhibit higher-order sub-Poissonian statistics.

2. HIGHER-ORDER SUB-POISSONIAN STATISTICS

For second-order sub-Poissonian statistics variance $\langle(\Delta m)^2\rangle = \langle m^2\rangle - \langle m\rangle^2$ of the photon-counting distribution is less than the mean of the distribution, $\langle m\rangle$. By noting that second-order factorial moment $\langle m^{(2)}\rangle \equiv \langle m(m-1)\rangle = \langle m^2\rangle - \langle m\rangle$, we can write the criterion for the second order sub-Poisson statistics as

$$\langle m^{(2)}\rangle - \langle m\rangle^2 < 0. \quad (1)$$

Note that for a Poissonian distribution $\langle m^{(2)}\rangle = \langle m\rangle^2$, so inequality (1) holds as an equality. The departures from Poisson statistics are then characterized in terms of the Fano factor ($F = \langle(\Delta m)^2\rangle/\langle m\rangle$) or the Q parameter $\{Q = [\langle m^{(2)}\rangle - \langle m\rangle^2]/\langle m\rangle\}$.[3,4] For a sub-Poissonian distribution the Q parameter is negative. We now extend this criterion to higher-order factorial moments. The lth (l is a positive integer) order factorial moment of the photocount distribution is defined by

$$\langle m^{(l)}\rangle = \sum_{m=l}^{\infty} m(m-1)\ldots(m-l+1)p(m,T), \quad (2)$$

where $p(m, T)$ is the probability of detecting m photons in the counting interval $[0-T]$. Here we have suppressed the time argument in the factorial moments. To extend the criteria for sub-Poissonian statistics to higher-order moments we introduce a parameter S_l:

$$S_l = \frac{\langle m^{(l)}\rangle}{\langle m\rangle^l} - 1. \quad (3)$$

It is easily proved that, for a Poisson distribution, $S_l = 0$ for all l. Parameter S_l for $l \geqslant 2$ provides a measure of the deviation of the lth factorial moment from that for a Poisson distribution with the same mean. $S_l > 0$ defines a super-Poissonian distribution, and $S_l < 0$ defines a sub-Poissonian distribution. Note that parameter S_2 is not equal to the Q parameter but is related to it by $S_2 = Q/\langle m\rangle$. As the values of higher-order factorial moments can be large, it is convenient to use normalized factorial moments rather than the analogs of Q to extend the concept of sub- and super-Poissonian statistics to higher-order moments. Another advantage of using the S_l parameters is that they are independent of the efficiency of detection.

We now show that negative values of parameter S_l for $l > 2$ (higher-order sub-Poissonian statistics) indicate the nonclassical nature of light. To establish this we note that for a classical field the factorial moments must satisfy the inequality[8,9]

$$\langle m^{(l+k)}\rangle\langle m^{(j-k)}\rangle \geqslant \langle m^{(l)}\rangle\langle m^{(j)}\rangle, \quad (4)$$

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Measurements of intensity fluctuations in a laser with a saturable absorber

Mansour Mortazavi,* Fuad Rawwagah, and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701
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Fluctuations of light intensity in a laser with an intracavity nonlinear absorber are studied experimentally. Measurements are made close to the tricritical point by varying the operating parameters of the laser. Experimental results are in agreement with the theoretical predictions based on the nonlinear oscillator model of the laser in which both the third- and fifth-order nonlinearities in the field amplitude are retained.

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I. INTRODUCTION

It is well known that the light from a single-mode laser operating below the threshold of oscillation where the nonlinearity of light-matter interaction plays a negligible role has the character of a narrowband thermal light. As the laser approaches and passes the threshold of oscillation, the nonlinearity of light-matter interaction becomes important. As a result of this nonlinearity, the field amplitude stabilizes at a nonzero value, relative intensity fluctuations tend toward zero, and the laser light approaches a coherent state with increasing excitation [1]. This transformation of laser light from a thermal state to a coherent state is likened to a second-order phase transition and can be described in terms of a third-order (in field amplitude) nonlinearity [2].

The physical mechanism responsible for the third-order nonlinearity in the laser is gain saturation. When a saturable absorber is added to the laser cavity, the nature of nonlinearity changes because the gain and absorption saturate, in general, at different rates. This leads to the appearance of new nonlinear phenomena such as bistability, hysteresis, and novel phase-transition analogies [3–5]. The type of fluctuation phenomena that the laser with a saturable absorber (LSA) can exhibit is extremely rich when both the atomic and field degrees of freedom are involved. Here we restrict ourselves only to class-A lasers where the atomic polarization and population can follow the field amplitude adiabatically. Under these conditions, the atomic variables can be eliminated from the equations of motion leaving the field amplitude as the only dynamical variable. These conditions are satisfied, for example, for a low-power He:Ne laser.

Intensity fluctuations for class-A LSA have been studied theoretically by a number of workers [4–9]. Experimental results for some special cases have also been reported [10]. In this paper, we present a systematic experimental investigation of the intensity fluctuations in the LSA. In our experiment, we are able to control the nature of nonlinearity and study laser light fluctuations as excitation is increased from below to above threshold. In Sec. II, we review the theoretical model that describes light-intensity fluctuations in class-A LSA. Section III describes the experimental procedure. Finally, Sec. IV compares experimental results with theoretical predictions.

*Permanent address: Department of Chemistry and Physics, University of Arkansas at Pine Bluff, Pine Bluff, AR 71601.

II. THEORY

We consider a single-mode laser containing an intracavity saturable absorber. We assume both the gain and absorber media to be a collection of two-level atoms in gas phase with strong Doppler broadening of atomic lines. In the gain medium, the active atoms are "pumped" to the excited state and in the absorber cell the active atoms are pumped to the lower level. Then for small pump levels, the equation of motion for the slowly varying dimensionless complex field amplitude $\mathcal{E}(t)$ of the LSA is [7,8,10]

$$\dot{E}(t)=E[a+b|E|^2-|E|^4]+q(t), \tag{1}$$

where the pump and saturation parameters a and b are given by

$$a=(1-\alpha-\mathcal{C}/\mathcal{A})\left[\frac{8\mathcal{A}^2}{3\mathcal{B}^2(\alpha s^2-1)}\right]^{1/3}, \tag{2}$$

$$b=(\alpha s-1)\left[\frac{8\mathcal{A}}{3\mathcal{B}(\alpha s^2-1)^2}\right]^{1/3}, \tag{3}$$

and $q(t)$ is a complex Gaussian white-noise process with zero mean and variance $\langle q^*(t_1)q(t_2)\rangle=4\delta(t_1-t_2)$. Here $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ are the gain, self-saturation, and loss parameters of Scully and Lamb theory [11,12]. Parameters with a bar, $\bar{\mathcal{A}}$ and $\bar{\mathcal{B}}$, refer to the absorption and self-saturation coefficients for the absorber. $\alpha=\bar{\mathcal{A}}/\mathcal{A}$ is the ratio of gain and absorption parameters and $s=(\bar{\mathcal{B}}/\bar{\mathcal{A}})/(\mathcal{B}/\mathcal{A})$ is the ratio of the saturation intensities for the gain and absorber atoms. Equation (1) was derived by using a perturbative approach to calculate the atomic polarization. For the LSA, terms up to fifth order in the field are required to ensure the establishment of a steady state for $s>1$ [5]. Equations (2) and (3) indicate that for fixed $\mathcal{A}$ and s the pump parameter a can be varied by changing the loss $\mathcal{C}$ and that, for fixed s and $\mathcal{A}$, the parameter b can be varied by changing the discharge current in the absorption cell. This changes $\bar{\mathcal{A}}$ and b via the relation $\alpha=\bar{\mathcal{A}}/\mathcal{A}$. In this way, we can explore the intensity fluctuations for the LSA in the entire threshold region.

Entanglement, Interference, and Measurement in a Degenerate Parametric Oscillator

Hua Deng, Daniel Erenso, Reeta Vyas, and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701
(Received 25 August 2000)

Quantum dynamical equations of motion for homodyne detection of the degenerate optical parametric oscillator are solved exactly. Nonclassical photon statistics are shown to be a consequence of interference of probability amplitudes, entanglement of photon pairs from such an oscillator, and the role of measurement in quantum evolution.

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Fluctuations of photon beams reflect the quantum dynamics of photoemissive sources. In quantum mechanics, probabilities for observed events are derived from an underlying wave function that can interfere and collapse as it evolves. A consequence of this is that quantum mechanics can lead to correlations between observed events which a classical stochastic theory may not. Examples of these nonclassical correlations include squeezing, antibunching, and violations of Bell's inequalities [1,2].

The subthreshold degenerate parametric oscillator (DPO) has played a central role in the study of nonclassical photon correlations, particularly, squeezing [1,2]. The DPO radiates a highly bunched light beam that exhibits a large degree of squeezing. Interestingly, the squeezed and highly bunched light from the DPO when combined with a coherent light field, as in homodyne detection, is expected to display a rich variety of nonclassical photon correlations including antibunching [3]. It is intriguing that a highly bunched entangled photon beam from the DPO when mixed with a coherent field will exhibit correlations similar to those exhibited by a single-atom resonance fluorescence in free space or in cavity quantum electrodynamics (QED) [4,5]. Antibunching of light emitted by a single two-level atomic system can be eventually traced to the atomic dead time that a two-level atom cannot emit a second photon immediately after the emission of a first photon. The situation is not so simple for homodyne detection of the light from the DPO because there is no obvious mechanism for a dead time. By solving the equations of motion for homodyne detection exactly, we show that nonclassical photon correlations in homodyne detection of the DPO are a consequence of the interference of probability amplitudes, entangled nature of photon pairs generated by the DPO, and measurement. These are the features that most distinguish quantum mechanics from classical mechanics.

An outline of the experimental setup for homodyne detection of the DPO light is shown in Fig. 1. The DPO and local oscillator (LO) fields are combined by a beam splitter to produce the source field at the detector. The field from the DPO is governed by the interaction Hamiltonian for a phase matched DPO driven by a classical injected signal of amplitude ε [6]:

$$\hat{H} = \frac{i\hbar\kappa\varepsilon}{2}(\hat{a}_d^{\dagger 2} - \hat{a}_d^2) + \hat{H}_{\text{loss}}. \tag{1}$$

Here κ is the mode-coupling constant and $\hat{a}_d$ and $\hat{a}_d^\dagger$ are the annihilation and creation operators, respectively, for the DPO. $\hat{H}_{\text{loss}}$ describes the loss suffered by the DPO field. The combination $\kappa\varepsilon$ can be chosen to be real by a suitable definition of phases.

The equation of motion for the density matrix $\hat{\rho}_d$ of the DPO field is then

$$\dot{\hat{\rho}}_d = \frac{\kappa\varepsilon}{2}[\hat{a}_d^{\dagger 2} - \hat{a}_d^2, \hat{\rho}_d] + \gamma(2\hat{a}_d\hat{\rho}_d\hat{a}_d^\dagger - \hat{a}_d^\dagger\hat{a}_d\hat{\rho}_d - \hat{\rho}_d\hat{a}_d^\dagger\hat{a}_d), \tag{2}$$

where 2γ is the cavity decay rate. The steady-state solution to this equation in positive-P representation is given by [6,7]

$$(\hat{\rho}_d)_{ss} = \frac{1}{\sqrt{2\bar{n}_d}\,\pi}\iint dx\, dy \frac{|x\rangle\langle y|}{\langle y\,|\,x\rangle} \times \exp\left[2xy - \frac{\gamma}{\kappa\varepsilon}(x^2 + y^2)\right], \tag{3}$$

where $-\infty < x, y < \infty$ are both real variables and $|x\rangle$ is a coherent state of $\hat{a}_d$ with $\hat{a}_d|x\rangle = x|x\rangle$. From this expression for the density matrix, the steady-state expectation value of an operator $\hat{O}$ can be calculated as $\langle\hat{O}\rangle_{ss} = \text{Tr}(\hat{O}\hat{\rho}_{ss})$. This leads to the following expectation values

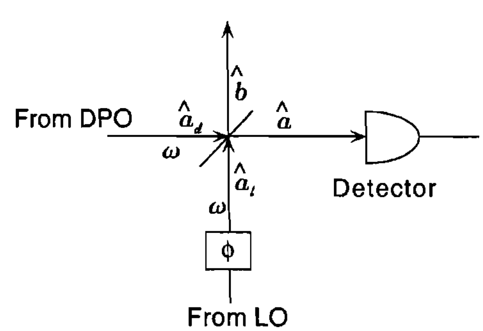

FIG. 1. Schematic experimental setup for the homodyne detection of the light from a degenerate parametric oscillator.

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Nonclassical effects in photon statistics of atomic optical bistability

Daniel Erenso, Ron Adams, Hua Deng, Reeta Vyas, and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701
(Received 30 May 2001; published 13 September 2001)

Homodyne statistics of light generated by an atomic system exhibiting optical bistability are analyzed. Using the dynamical equations of motion for a single atom in a coherently driven cavity in the good cavity limit, we show that the homodyne field can be described in terms of two independent real Gaussian stochastic processes and a coherent component. By making a Karhunen-Loève expansion of the field variables we derive the generating function for the photoelectron statistics. From this generating function photoelectron-counting distribution, factorial moments, and waiting-time distribution are obtained analytically. These quantities are directly measurable in photon-counting experiments. We show that the homodyne field exhibits many interesting nonclassical features including nonclassical effects in higher-order factorial moments.

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I. INTRODUCTION

Interaction of a single two-level atom with a quantum field inside a coherently driven cavity in the good cavity limit, is known to show optical bistability [1,2]. We will refer to this system as single atom optical bistability (SAOB). Similarly, N two-level atoms placed inside a coherently driven cavity also exhibit optical bistability that we shall refer to as multiatom optical bistability (MAOB) [3,4]. These systems are also known to show antibunching, although the size of antibunching is small. In order to enhance antibunching and other nonclassical effects, several schemes based on interference [5], passive filter cavities [6], or homodyne detection [7] have been proposed.

Homodyning a field with a coherent local oscillator (LO) provides one way of enhancing nonclassical effects. The homodyne field can exhibit strong nonclassical features, which are not shown by the original field. The homodyne statistics are sensitive to the phase difference between the signal and the LO. An example of this behavior is provided by the light from the degenerate parametric oscillator, which is highly bunched and super-Poissonian. When this field is homodyned with a LO, the homodyne field shows a variety of nonclassical effects such as antibunching, sub-Poissonian statistics, and violation of other classical inequalities [8–10].

In this paper we consider homodyning of the light from a system that exhibits SAOB with the light beam from a LO at a lossless beam splitter as shown in Fig. 1. A detector of efficiency η placed at one of the output ports of the beam splitter detects the homodyne field and generates photoelectric pulses, which are measured by suitable electronics. We study photoelectron statistics measured by the detector. In Sec. II we start from the equations of motion derived by Wang and Vyas for a single two-level atom in the good cavity limit [2] and show that the field from the SAOB can be expressed in terms of two Gaussian random variables. We then derive the equations that govern the dynamics of the homodyne field. Using these equations and applying the Karhunen-Loève expansion for the field variables, we calculate the moment generating function for the photocount distribution. We also show that a system exhibiting MAOB can also be described by similar expressions with an appropriate change in parameters. In Sec. III we present an analytic expression for the moment generating function. Photon statistics of the homodyne field are then analyzed with the help of the moment generating function. The photocount distribution, its moments, and the waiting-time distribution for the homodyne field are calculated. Finally, in Sec. VI, a summary of the main results of the paper is presented.

II. DYNAMICS OF THE HOMODYNE FIELD AND THE GENERATING FUNCTION

In this section we derive equations of motion describing the dynamics of the homodyne field when the signal is from the SAOB. We will see that similar equations are obtained when the signal is from the MAOB.

Consider a single damped two-level atom with transition frequency ω_a, interacting with a single mode of a cavity with resonance frequency ω_c. The cavity is driven by a coherent external field of frequency ω_o and amplitude ϵ. In the electric dipole and rotating-wave approximation, the Hamiltonian for this system can be written as

$$\hat{H} = \omega_a \hat{\sigma}_z + \hbar \omega_c \hat{a}^\dagger \hat{a} + ig\hbar(\hat{a}^\dagger \hat{\sigma}_- + \hat{a}\hat{\sigma}_+) + i\hbar\epsilon(\hat{a}^\dagger e^{-i\omega_o t} - \hat{a} e^{i\omega_o t}) + \hat{H}_{loss}. \quad (1)$$

Here $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators for the cavity mode, $\hat{\sigma}_+$, $\hat{\sigma}_-$, and $\hat{\sigma}_z$ are the Pauli spin matrices describing the two-level atom, g is the atom-field coupling constant, and $\hat{H}_{loss}$ describes atomic losses due to spontaneous decay and field losses at the cavity mirrors.

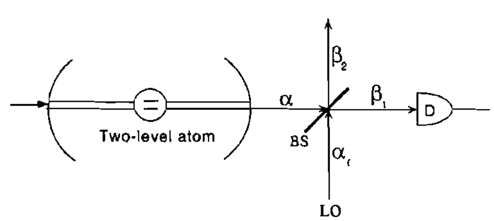

FIG. 1. System for homodyning the SAOB field with the LO field. BS denotes the beam splitter and D denotes a detector.

Higher-order nonclassical effects in a parametric oscillator

Reeta Vyas and Surendra Singh
Physics Department, University of Arkansas, Fayetteville, Arkansas 72701
(Received 10 January 2000; published 8 August 2000)

Factorial moments for the light from a degenerate parametric oscillator mixed with a coherent local oscillator are calculated and shown to reveal novel nonclassical features of light. We have found novel regimes where the nonclassical character of the field is reflected not in the violations of inequalities based on the second-order moments but in those based on higher order moments. These violations can be observed in photoelectric measurements of light.

PACS number(s): 42.50.Dv, 42.50.Ar, 42.65.Ky

Nonclassical properties of light such as squeezing [1], sub-Poissonian statistics [2], and antibunching [3] have been discussed in terms of quadratic field or intensity correlations. With the development of techniques for making higher order correlation measurements, the interest in nonclassical effects naturally extends to the higher order correlations. Of particular interest are physical systems where nonclassical features of light are revealed in higher order correlations but not in the lower order correlations. Higher order squeezing was introduced by Hong and Mandel [4]. Agarwal and Tara introduced nonclassical behavior in terms of normally ordered moments of a distribution [5]. Lee [6] considered a single mode field and defined higher order nonclassical effects in terms of factorial moments of the photon distribution by using majorization theory.

In this paper we present a simple criterion for higher order nonclassical behavior in terms of factorial moments of the photoelectron counting distribution. We then show that the light from a subthreshold degenerate parametric oscillator (DPO), when superposed on the light from a coherent local oscillator (LO), satisfies the criteria for higher order nonclassical behavior in terms of factorial moments. These violations can be measured in photon counting experiments.

Let us first recall that the probability of detecting m photoelectrons in the time interval $[0-T]$ is [7]

$$p(m,T)=\left\langle :\frac{\hat{U}^m}{m!}e^{-\hat{U}}:\right\rangle \tag{1}$$

where the colons $:\cdots:$ denote time ordering and normal ordering of the operator product between the colons. In writing Eq. (1), we have assumed stationary light fields. The operator $\hat{U}$ is given by

$$\hat{U}=\eta\int_0^T\hat{I}(t)dt, \tag{2}$$

where $\hat{I}(t)$ is the photon flux operator (number of photons per second) and η is the detection efficiency. The angular brackets denote averaging with respect to the state of the field.

The ℓ-th order factorial moment (ℓ is a positive integer) of the photoelectron counting distribution is defined by

$$\langle m^{(\ell)}\rangle=\sum_{m=1}^{\infty} m(m-1)\cdots(m-\ell+1)p(m,T), \tag{3}$$

where, for simplicity, we have suppressed the dependence of the moments on the counting interval T. The second factorial moment $\langle m^{(2)}\rangle$ is related to the variance $\langle(\Delta m)^2\rangle$ of photon number distribution by

$$\langle(\Delta m)^2\rangle=\langle m^{(2)}\rangle-\langle m\rangle^2+\langle m\rangle. \tag{4}$$

For a coherent field (Poissonian distribution) $\langle(\Delta m)^2\rangle=\langle m\rangle$ is independent of the value of T. Expressing $p(m,T)$ in Eq. (3) in terms of $\hat{U}$ and using the Glauber-Sudarshan P representation for the density matrix of the field, the factorial moments can be written as

$$\langle m^{(\ell)}\rangle=\langle:\hat{U}^{\ell}:\rangle=\int U^{\ell}P(U)dU, \tag{5}$$

where U is a positive real number and for a classical state the probability distribution $P(U)$ must be a positive and nonsingular function (no more singular than a δ function). Now define a positive function in terms of two positive real variables U and W as

$$\sum_{p=j}^{\ell} U^{(\ell+j-p-1)}W^{(p-1)}(U-W)^2\geq 0. \tag{6}$$

On expanding and simplifying this, we find that

$$(U^{(\ell+1)}W^{(j-1)}+U^{(j-1)}W^{(\ell+1)})\geq(U^{(\ell)}W^{(j)}+U^{(j)}W^{(\ell)}). \tag{7}$$

Therefore for a classical probability density $P(U,W)$ $[\equiv P(U)P(W)]$, which is positive and no more singular than a δ function,

$$\int P(U,W)(U^{(\ell+1)}W^{(j-1)}+U^{(j-1)}W^{(\ell+1)})dU\,dW$$
$$\geq\int P(U,W)(U^{(\ell)}W^{(j)}+U^{(j)}W^{(\ell)})dU\,dW. \tag{8}$$

Thus the factorial moments of photon counting distribution for a classical field must satisfy the inequalities

Antibunching and photoemission waiting times

Reeta Vyas and Surendra Singh

Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701

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We discuss antibunching of photons in terms of the distributions of waiting between successive photoemissions and compare it with definitions of antibunching based on the two-time intensity correlation function. We illustrate our results for photon sequences emitted by parametric oscillators. Curves are presented to illustrate the behavior. © 2000 Optical Society of America [S0740-3224(00)02304-3]

OCIS codes: 270.0270; 270.5290; 270.2500; 190.4970.

Nonclassical properties of the electromagnetic field have continued to attract great attention, as they provide a testing ground for the predictions of quantum electrodynamics. Nonclassical properties of the electromagnetic field are reflected in squeezing,[1] sub-Poissonian statistics,[2,3] antibunching,[4–7] and violation of various classical inequalities.[6–10] These nonclassical effects refer to different aspects of the field. For example, squeezing refers to the wavelike character of the field. It is measured in an interference experiment. Antibunching and sub-Poissonian statistics, on the other hand, refer to the particlelike character of the field and are measured in photoelectric counting experiments.

Antibunching was one of the first nonclassical features of the electromagnetic field to be observed experimentally.[4] It refers to the tendency of photons to be separated from one another in time. Bunching refers to the opposite tendency of photons to be bunched together in time. Whether a photon sequences exhibits bunching or antibunching is intimately connected to the source dynamics. Figure 1 shows three different photon-emission sequences. Each vertical line represents a photoemission event. We can also think of these sequences as photodetection events at the output of an ideal (unit detection efficiency) photoelectric detector. For an ideal detector the distinction between a photodetection and a photoemission sequence is not important. In what follows we shall assume this to be the case and speak of photoemissions and photoelectrons interchangeably. Furthermore, for simplicity, we shall restrict ourselves to stationary sequences. All sequences in Fig. 1 have the same average rate of occurrence of photons. Sequence (a) is a random (uncorrelated) photoemission sequence such as that which might be generated by a laser operating high above threshold. Such a sequence is also called a Poisson sequence. A comparison of sequences (a) and (b) shows that photons in sequence (b) tend to bunch together. This is an example of a bunched photon sequence that might be generated by a thermal source or a laser operating far below threshold. In sequence (c), photons tend to be separated from one another. This is an example of an antibunched photon sequence that might be generated, for example, by a fluorescing single two-level atom.

The physical picture of photon bunching and antibunching developed in Fig. 1 can be quantified in terms of the distribution of waiting times between successive photoemissions or photodetections. This disruption is given by[10–14]

$$w(T) = \frac{\langle \mathcal{T}{:}\hat{I}(t)\{\exp[-\int_t^{t+T} \hat{I}(t')\mathrm{d}t']\}\hat{I}(t+T){:}\rangle}{\langle \hat{I}\rangle}, \quad (1)$$

where $\hat{I}(t)$ is the intensity (photon flux from the source in units of number of photons per second) operator at time t and $\langle \mathcal{T}{:}\ {:}\rangle$ stands for time ordering and normal ordering of the operator product between the colons. Note that $w(T)$ involves the detection of two successive photons at times t and $t+T$ and of no photons in the interval $(t, t+T)$. The probability of observing an interval between T and $T+\mathrm{d}T$ between successive photoemissions is $w(T)\mathrm{d}T$. The waiting-time distribution $w(T)$ refers to the separation between photons and provides a clear physical picture of photon bunching and antibunching in the time domain.[11,13,14]

Waiting times for coherent light (sequence of random photons) are exponentially distributed according to $w_c(T) = \langle \hat{I}\rangle \exp(-\langle \hat{I}\rangle T)$, where $\langle \hat{I}\rangle$ is the average intensity (photon flux from the source). It is clear that $w_c(0)/\langle \hat{I}\rangle = 1$. The average separation between successive photons is $1/\langle \hat{I}\rangle$, and the most probable waiting time is zero. In an antibunched photon sequence, photons tend to be less bunched in time than photons in a random photon sequence. This means that, for an antibunched photon sequence, zero waiting time is less probable than for a random photon sequence. This leads to the criterion

$$w(0) < w_c(0) \quad (2)$$

for photon antibunching in terms of the waiting-time distribution. Similar considerations hold for a bunched photon sequence, in which photons tend to be more bunched than in a random photon sequence; zero waiting time will be more probable than for a Poisson sequence. Thus for a bunched photon sequence $w(0) > w_c(0)$.

The criterion for photon antibunching in terms of $w(T)$ can be related to the traditional criteria in terms of the normalized second-order intensity correlation function $g^{(2)}(T)$. This correlation function is the joint probability

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Two-photon detection of light from a degenerate parametric oscillator

S. Bali, K.F. Lee, Surendra Singh *

Department of Physics, University of Arkansas, Fayetteville, AR 72701, USA

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Abstract

Photocount statistics of light emitted by a degenerate parametric oscillator (DPO) are analyzed using a two-photon detection scheme in the limit where the counting interval is much shorter than the correlation time of the light field. Results are compared to the usual one-photon detection scheme. Counting distributions for the DPO are also compared with those of thermal light and laser light. © 1997 Elsevier Science B.V.

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1. Introduction

Measurements of higher-order photon correlations of light beams have been of interest to workers in the quantum optics community for quite sometime [1,2]. The accuracy of these measurements, all of which were based on single-photon detection schemes, were limited by detector and electronic dead-time effects and by the requirement for small photon fluxes. Recently, a scheme based on two-photon detection was described [3,4] that partly overcame dead-time limitations. This scheme can handle relatively large as well as low photon fluxes thus illustrating the advantages of a two-photon detector. It has also been noted that two-photon detection may provide a more sensitive probe of the fluctuations of the field, in the sense that the rth factorial moment of the two-photon counting distribution reflects $2r$th order intensity correlation function instead of the rth order intensity correlation function in the case of single-photon counting [5,6]. Furthermore, information inaccessible by single-photon counting, when the density operator of the field is not factorizable into a product of single-mode density operators [7], can also be extracted.

A two-photon detector is a device that outputs a pulse only upon the simultaneous absorption of two photons [8]. Such a two-photon detector can be constructed, for instance, by placing a nonlinear crystal in the path of the light beam and then detecting the second-harmonic signal with a single-photon detector [3,4,8–11]. Obviously, the statistics of the second harmonic photons as recorded by a single-photon detector, are nothing but the statistics of the fundamental as recorded by a two-photon detector.

On the other hand, workers in the optics community have also been interested for some time in the unusual statistical properties of the light emitted by the optical parametric oscillator when operating below threshold [12–14]. The purpose of this paper is to investigate what two-photon counting would yield on the light output of a degenerate (i.e. the twin down converted beams have the same frequency) parametric oscillator (DPO).

We begin with a brief review in Section 1 of earlier work [5,6] on thermal and laser fields. In Section 2 we describe two-photon detection of the output of a DPO operating below threshold. Section 3 is a discussion of the results obtained in the preceding sections.

* Corresponding author.

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Homodyne detection for the enhancement of antibunching

Reeta Vyas, Changxin Wang, and Surendra Singh
Physics Department, University of Arkansas, Fayetteville, AR 72701
(Received 15 February 1996; revised manuscript received 22 April 1996)

We propose a scheme based on homodyne detection for enhancing antibunching in second-harmonic generation and multiatom optical bistability. We show that depending on the reflectivity of the beam splitter, relative field strengths, and relative phase it is possible to achieve perfect antibunching in the superposed field. We also discuss other nonclassical effects exhibited by the superposed field and present curves to illustrate the behavior. [S1050-2947(96)09008-7]

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I. INTRODUCTION

Squeezing [1], antibunching, and sub-Possonian statistics [2,3] are nonclassical features of the electromagnetic field. These nonclassical features have been of considerable interest as they provide testing grounds for the prediction of quantum electrodynamics. Squeezing is related to the wavelike character of the electromagnetic field. It is measured in interference experiments. Antibunching and sub-Poissonian statistics, however, reflect the particlelike behavior of the field and are measured in photon counting experiments. As discussed in Ref. [4] squeezing, antibunching, and sub-Poissonian statistics are, in general, distinct nonclassical effects in the sense that an electromagnetic field may exhibit one but not the other.

The antibunching effect has been predicted in intracavity second-harmonic generation (ISHG) [5,6] and multiatom optical bistability (MAOB) [7,8]. However, the predicted size of antibunching is small and would be difficult to detect experimentally, as it occurs against a large coherent background. The predicted antibunching in these systems is inversely proportional to the saturation photon number n_O, which is of the order of 10^6-10^8 for the ISHG, and 10^3-10^4 for the MAOB. Several schemes based on interference [9] or passive filter cavities [10–12] have been proposed to enhance the antibunching effect.

We propose a scheme based on homodyne detection [13–15] for enhancing antibunching in these systems. Homodyne detection experiments have been used for measuring phase-sensitive properties of squeezed light [1]. It has been shown that the light from a degenerate parametric oscillator, which is highly bunched and super-Poissonian [16,17], can exhibit many nonclassical effects using a similar detection scheme [13]. In the homodyne detection experiment we consider the interference of the signal beam from the ISHG or the MAOB with a coherent local oscillator (LO) at a lossless beam splitter as shown in Fig. 1. A detector of efficiency η is placed at one of the output ports of the beam splitter. The statistics measured at the detector is sensitive to the relative phase between the signal and the LO. Thus particlelike properties (photon statistics) are intimately connected to wavelike (phase) property of the field. Because of this phase dependence, the homodyne field can exhibit enhanced antibunching and violation of various classical inequalities. Since in this scheme one can readily adjust various parameters such as the strength of the local oscillator, transmittance, and relative phase, this scheme may provide a better way of enhancing antibunching.

In Sec. II we briefly describe the homodyne detection scheme. In Sec. III we apply this technique to the ISHG. In Sec. IV we discuss the enhancement of antibunching for the MAOB. Finally, a summary and main conclusions of the paper are presented in Sec. V.

II. HOMODYNE DETECTION

Figure 1 shows a schematic diagram for the homodyne detection experiment. For the ISHG, a nonlinear crystal is placed inside the cavity, whereas for the MAOB, N two-level atoms are placed inside the cavity. The light from the ISHG or the MAOB is superimposed with the light from a LO at a lossless beam splitter. The annihilation operators $\hat{b}_1$ and $\hat{b}_2$ at the output ports are related to those at the input ports by [13,14]

$$\begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = \begin{pmatrix} \sqrt{\mathcal{T}} & \sqrt{\mathcal{R}} \\ -\sqrt{\mathcal{R}} & \sqrt{\mathcal{T}} \end{pmatrix} \begin{pmatrix} \hat{a}_s \\ \hat{a}_\ell \end{pmatrix},$$

with

$$\mathcal{T}+\mathcal{R}=1 .$$

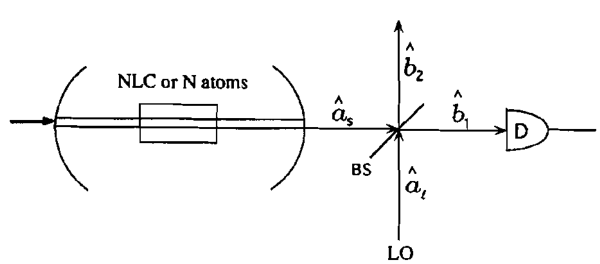

FIG. 1. System for homodyning the ISHG or MAOB field with the LO field. For the ISHG a nonlinear crystal (NLC) is placed inside the cavity and for the MAOB N two-level atoms are placed inside the cavity. BS denotes a beam splitter and D denotes a detector.

Measurements of Higher Order Photon Bunching of Light Beams

Yujiang Qu and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701

Cyrus D. Cantrell
Department of Physics, University of Texas, Dallas, Texas 75083
(Received 1 November 1995)

A two-photon detection scheme is used to measure three- and four-photon correlations in a light beam and study their time dependence.

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Photon correlations of light beams are intimately connected to source dynamics. They are characteristic of sources that produce them [1]. Thus while photons from thermal sources exhibit bunching [2], those from nonthermal sources may exhibit antibunching [3]. Photon correlations are usually discussed in terms of the second order intensity correlation function $\langle I(t)I(t+\tau)\rangle$, where $I(t)$ is the intensity of light beam and angular brackets indicate averaging with respect to the state of the field. The second order correlation function is proportional to the probability of detecting two photons separated by an interval τ. It is clear that even for uncorrelated photons there is some finite probability, proportional to $\langle I(t)\rangle\langle I(t+\tau)\rangle = \langle I\rangle^2$, of detecting a pair of photons separated by an interval τ. A measure of intrinsic two-photon bunching is therefore provided by the correlation function $\kappa_2(\tau) = \langle \Delta I(t)\Delta I(t+\tau)\rangle/\langle I\rangle^2$, where $\Delta I(t) = I(t) - \langle I\rangle$ represents deviations of light intensity from the mean. Here and in what follows we assume statistically stationary light beams. Similar to $\kappa_2(\tau)$, we can introduce higher order correlation functions of light.

The third order intensity correlation function $\langle I(t)I(t+\tau_1)I(t+\tau_1+\tau_2)\rangle$ is proportional to the probability of detecting three photons at times t, $t+\tau_1$, and $t+\tau_1+\tau_2$, respectively. Coincidences of third order include effects of three-photon chance coincidences, proportional to $\langle I\rangle^3$, and two correlated photons in chance coincidence with a third one, proportional to $\langle I(t)I(t+\tau_1)\rangle\langle I(t+\tau_1+\tau_2)\rangle$, etc. After subtracting these contributions, we find the intrinsic third order correlations are given by $\kappa_3(\tau_1,\tau_2) = \langle \Delta I(t)\Delta I(t+\tau_1)\Delta I(t+\tau_1+\tau_2)\rangle/\langle I\rangle^3$. Similarly, the intrinsic fourth order correlations are given by the function $\kappa_4(\tau_1,\tau_2,\tau_3) = \langle \Delta I(t)\Delta I(t+\tau_1)\Delta I(t+\tau_1+\tau_2)\Delta I(t+\tau_1+\tau_2+\tau_3)\rangle/\langle I\rangle^4 - \kappa_2(\tau_1)\kappa_2(\tau_3) - \kappa_2(\tau_1+\tau_2)\kappa_2(\tau_2+\tau_3) - \kappa_2(\tau_1+\tau_2+\tau_3)\kappa_2(\tau_2)$ [4]. The zero-delay correlations $\kappa_3(0,0)$ and $\kappa_4(0,0,0)$ describe the degree of intrinsic three- and four-photon bunching, respectively. The zero-delay intensity correlations were measured in counting experiments based on single-photon detectors by Chang and co-workers [5]. The accuracy of these measurements of higher order correlations is limited by the detector and electronic dead-time effects. This limitation becomes especially significant for fields that exhibit large intensity fluctuations [5,6]. Ironically, it is precisely for this type of fields that measurements of higher order correlations provide information that cannot be inferred from knowledge of lower order correlations.

Counting experiments do not allow us to determine the time dependence of intensity correlations. The time dependence of intensity correlations is measured in delayed coincidence experiments [7–10]. Counting experiments also require large count rates for measuring higher order moments, whereas correlation measurements can be carried out with relatively low (at least by a factor of 10^{-2}) count rates. The measurements of the time dependence of the third order correlation function were studied by several workers [9,10] using single-photon detection schemes. These experiments clearly underscore the increasing difficulty of measuring the time dependence of intensity correlations higher than the second order by these techniques.

In this paper we wish to describe a scheme based on two-photon detection of light that allows us to measure third and fourth order intensity correlations. This scheme partly overcomes dead-time limitations and involves the measurements of autocorrelation and cross-correlation functions of the second harmonic and the fundamental field. We demonstrate our method by measuring the third and fourth order correlation functions of photons in a laser near threshold. Our experiments also yield the correlation times over which three- and four-photon correlations persist. The method is applicable to a wide range of experiments where higher order correlations of light play an important role.

Consider the generation of second harmonic (SH) light from a fundamental beam. The intensity of the SH beam $I_2(t)$ is related to the intensity $I(t)$ of the fundamental beam by

$$I_2(t) = \text{const} \times I^2(t)\,. \tag{1}$$

Using this relation in the cross-correlation function $C(\tau) = \langle I_2(t)I(t+\tau)\rangle/\langle I_2\rangle\langle I\rangle$ of the second harmonic and fundamental light intensity, we immediately see that

$$C(\tau) = \frac{\langle I^2(t)I(t+\tau)\rangle}{\langle I^2\rangle\langle I\rangle} \tag{2}$$

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Measurements of photon statistics in second-harmonic generation

Yujiang Qu and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701
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Statistical properties of second-harmonic photons generated by a single-mode laser operating near threshold are investigated by means of photoelectric counting measurements. It is found that second-harmonic photons display superthermal fluctuations below threshold where pump statistics are thermal. Above threshold, as the pump field becomes coherent, second-harmonic photons approach Poisson statistics. These measurements, interpreted as two-photon statistics of the fundamental beam, provide experimental evidence for the dependence of the fluctuation properties of a light beam on the process of detection.

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I. INTRODUCTION

Optical second-harmonic generation (SHG) is a two-photon process. It involves the absorption of two photons from an optical beam of frequency ω_1 (the fundamental) followed by the generation of a single photon of frequency $\omega_2 = 2\omega_1$ (the second harmonic). In order to conserve energy and momentum, second-harmonic generation can occur only in certain anisotropic nonlinear media that lack inversion symmetry [1]. Since second-harmonic generation depends on the absorption of two photons, it is sensitive to photon correlations of the fundamental beam. This has indeed been observed in a number of experiments [2–5]. In these experiments thermal or multimode fundamental beams were used to generate a second-harmonic signal. It was found that, for a fixed mean light intensity of the fundamental beam, more of the second-harmonic signal was generated with beams that exhibited excess two-photon bunching than with a coherent fundamental beam of the same mean intensity. The effect of SHG is to selectively remove pairs of photons from the fundamental beam. Evidently this will change the statistics of the fundamental beam. However, this change in the properties of the fundamental beam will be small unless optical cavities are present to provide feedback at the second harmonic and the fundamental frequencies [6]. Although the statistical properties of the second-harmonic beam will depend on the statistical properties of the fundamental beam, in general, the two will be different from each other. Because of the two-photon character of the second-harmonic generation process, the statistics of second-harmonic photons will be a nonlinear transformation of the statistics of the fundamental photons. There is another way of looking at the second-harmonic photon statistics. The second-harmonic statistics is the statistics of the fundamental beam as measured by a two-photon detector [7]. According to this view, the statistics of second-harmonic photons are the statistics of the fundamental beam as recorded by a two-photon detector.

In this paper we describe measurements of photon statistics of the second-harmonic field when the fundamental field is derived from a single-mode laser operating near threshold. It is well known that a single-mode laser operating near threshold exhibits thermal statistics below threshold and Poisson statistics above threshold [8]. In the region of threshold it makes a transition from thermal to Poisson statistics. Thermal statistics are characterized by excess photon bunching, whereas Poisson statistics implies only random photon bunching. With such a fundamental pump, therefore, we can study the dependence of the second-harmonic statistics on the fundamental statistics under a variety of conditions. We begin with a review of the statistics of second-harmonic generation with a single-mode laser pump in Sec. II. In Sec. III we describe the photoelectric measurements of the second-harmonic photons. We conclude the paper with results and a discussion in Sec. IV.

II. STATISTICS OF SECOND-HARMONIC PHOTONS

Consider a fundamental beam derived from a single-mode laser operating near threshold. The distribution of light intensity for the fundamental field is then described by [8]

$$P_1(I_1) = \frac{1}{\mathcal{N}} \exp\left[\frac{1}{2}aI_1 - \frac{1}{4}I_1^2\right], \tag{1}$$

where I_1 is suitably scaled (dimensionless) intensity of the fundamental laser beam and a is the pump parameter of the laser. The pump parameter a is negative below threshold, positive above threshold, and zero at laser threshold ($a = 0$). The normalization constant $\mathcal{N}$ is given by

$$\mathcal{N} = \sqrt{\pi} e^{a^2/4} \mathrm{erfc}(-a/2), \tag{2}$$

where $\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$ is the complement of the error function [10,11]. Consider now the second harmonic generated by this fundamental field inside a nonlinear

Exact Quantum Distribution for Parametric Oscillators

Reeta Vyas and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701
(Received 15 August 1994)

An exact quantum distribution for the nondegenerate parametric oscillators is presented and used to discuss their coherence properties. It is found that while each mode individually approaches a classical state, many quantum features exhibited by their combination survive even in the semiclassical limit.

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Optical parametric oscillators (OPOs) are quantum mechanical devices with a definite threshold for self-sustained oscillations [1–4]. They have played a central role in squeezing and twin-beam noise reduction experiments [5,6]. Theoretical understanding of these properties of the OPOs has been based mostly on linearized treatments above and below the threshold of oscillation [7–9]. In the region of threshold where linearization fails, the complex-P distribution has been used [10]. The complex-P distribution, unfortunately, does not have the character of a probability density and is of limited use for gaining insights into the coherence properties of the OPOs. Another distribution, closely related to the complex P, is the positive-P distribution which is a true probability density [11]. In this paper we present an analytic solution for the positive-P function for the optical parametric oscillators. With the help of analytic solutions tremendous insights into the coherence properties of other oscillators have been gained [12–14]. Our analytic treatment is based on the observation that the quantum dynamics of OPOs is naturally confined to a bounded region in an eight-dimensional phase space. The solution presented here provides us with an elegant picture of how the coherence properties of the OPOs are transformed in the threshold region. It also allows us to discuss quantum features that survive even as the field amplitudes grow up to macroscopic values above threshold.

We model the parametric oscillator by two quantized field modes of frequencies ω_1 and ω_2 interacting with a third mode of frequency $\omega_3 = \omega_1 + \omega_2$ inside an optical cavity via a $\chi^{(2)}$ nonlinearity. Modes ω_1 and ω_2 experience linear losses characterized by the decay rates $\gamma_1 = \gamma_2 = \gamma$, and mode ω_3 suffers linear losses characterized by the decay γ_3. The cavity is excited by a classical pump at frequency ω_3. In the interaction picture the microscopic Hamiltonian takes the form

$$\hat{H} = i\hbar\kappa(\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{a}_3 - \hat{a}_3^\dagger\hat{a}_1\hat{a}_2) + i\hbar\gamma_3\epsilon(\hat{a}_3^\dagger - \hat{a}_3) + \hat{H}_{\text{loss}}, \tag{1}$$

where $\hat{a}_j$ and $\hat{a}_j^\dagger$ are the annihilation and creation operators for the modes, κ is the mode coupling constant, ϵ is the pump field amplitude, and $\hat{H}_{\text{loss}}$ describes mode losses.

This nonlinear quantum mechanical problem can be mapped into a classical stochastic process by using the positive-P representation [11]. Eliminating the pump mode adiabatically ($\gamma_3 \gg \gamma$) we obtain the following set of Ito stochastic differential equations:

$$\dot{\alpha}_1 = -\alpha_1 + \frac{\sigma}{n_0}\alpha_{2*} - \frac{2}{n_0}\alpha_1\alpha_2\alpha_{2*} + \frac{1}{\sqrt{n_0}} \times \sqrt{\sigma - 2\alpha_1\alpha_2}\,(\eta_1 + i\eta_2), \tag{2}$$

$$\dot{\alpha}_2 = -\alpha_2 + \frac{\sigma}{n_0}\alpha_{1*} - \frac{2}{n_0}\alpha_2\alpha_1\alpha_{1*} + \frac{1}{\sqrt{n_0}} \times \sqrt{\sigma - 2\alpha_1\alpha_2}\,(\eta_1 - i\eta_2), \tag{3}$$

$$\dot{\alpha}_{1*} = -\alpha_{1*} + \frac{\sigma}{n_0}\alpha_2 - \frac{2}{n_0}\alpha_{1*}\alpha_2\alpha_{2*} + \frac{1}{\sqrt{n_0}} \times \sqrt{\sigma - 2\alpha_{1*}\alpha_{2*}}\,(\eta_3 + i\eta_4), \tag{4}$$

$$\dot{\alpha}_{2*} = -\alpha_{2*} + \frac{\sigma}{n_0}\alpha_1 - \frac{2}{n_0}\alpha_{2*}\alpha_1\alpha_{1*} + \frac{1}{\sqrt{n_0}} \times \sqrt{\sigma - 2\alpha_{1*}\alpha_{2*}}\,(\eta_3 - i\eta_4), \tag{5}$$

where η_i are white noise Gaussian random processes with zero mean and correlation functions given by $\langle\eta_i(t)\eta_j(t')\rangle = \delta_{ij}\delta(t - t')$. Here time is measured in units of γ^{-1}, $n_0 = 2\gamma\gamma_3/\kappa^2$ is parameter that sets the scale for the number of photons necessary to explore the nonlinearity of interaction, and $\sigma = 2\gamma_3\epsilon/\kappa$ is a dimensionless measure of the pump field amplitude scaled to give $\sigma = n_0$ as the threshold condition. In the absence of mode losses adiabatic elimination of the pump mode is not justified [15]. Note that the adiabatic approximation does not amount to a neglect of the entanglement of pump and down-converted modes. Complex variables α_i and α_{i*} correspond to $\hat{a}_i$ and $\hat{a}_i^\dagger$, respectively. In the positive-P representation, α_i and α_{i*} are not complex conjugates of each other.

Equations (2)–(5) describe trajectories in an eight-dimensional phase space. An examination of these equations reveals that the eight-dimensional phase space is naturally divided into two subspaces. If we consider the four-dimensional subspace $\alpha_2 = (\alpha_1)^*$, $\alpha_{2*} = (\alpha_{1*})^*$, and $|\alpha_1|^2 < \sigma/2$, $|\alpha_{2*}|^2 < \sigma/2$, we notice that the trajectories starting in this subspace initially remain confined to this subspace. In other words, the condition $\alpha_2 = (\alpha_1)^*$, $\alpha_{2*} = (\alpha_{1*})^*$ is preserved for all times if initially we start out in this subspace. The initial state,

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Polarization properties of Maxwell-Gaussian laser beams

W. L. Erikson* and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701
(Received 10 December 1993)

Gaussian beam solutions of Maxwell's equations are constructed in terms of the solutions of the paraxial scalar wave equation. Explicit expressions for the field components of Hermite-Gaussian laser beams are given and their polarization and propagation characteristics are discussed. Experimental evidence for the polarization structure of Hermite-Gaussian laser beams is presented.

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I. INTRODUCTION

Laser beams are wavelike electromagnetic disturbances that have a predominant direction of propagation and a finite cross section transverse to the direction of propagation. These beams are commonly modeled by Hermite-Gaussian beams [1–3]. This is usually done within the framework of the scalar and paraxial approximations. For most applications which do not involve the polarization properties of laser beams, this framework is quite adequate. However, a scalar representation fails to describe the polarization properties of laser beams correctly. Indeed, a scalar description of finite cross-section laser beams is inconsistent with Maxwell's equations even for linearly polarized laser beams. The transverse character of the electromagnetic field, expressed by two of Maxwell's equations $\vec{\nabla}\cdot\vec{E}(\vec{r},t)=0$ and $\vec{\nabla}\cdot\vec{B}(\vec{r},t)=0$, implies that the spatial variation of the field in directions transverse to the direction of propagation is coupled to the polarization properties of the field. Thus it is well known that spatial variation of the field in transverse directions gives rise to a longitudinal field component [4–11].

The coupling of transverse spatial variation of nonplanar wave fronts to polarization was investigated in an interesting paper by Fainman and Shamir [6]. They analyzed the cross polarization in a spherical wave front from a point source. They also recorded experimentally the cross polarization of a linearly polarized fundamental Gaussian beam passing through a pin hole. Several other workers have also discussed the polarization properties of the fundamental Gaussian laser beam [7,8]. They show that the fundamental Gaussian beam will always exhibit cross polarization even without passing through a pin hole. The general problem of beamlike solutions of Maxwell's equations has also been treated by several workers [9,10] in terms of electromagnetic potentials.

In this paper we present a simpler and more direct approach based on the solutions of the paraxial wave equation. Using this approach we establish the general field (polarization) structure of paraxial beamlike solutions of Maxwell's equations. These expressions are then used to describe explicitly the polarization and propagation characteristics of the fundamental as well as higher order Hermite-Gaussian laser beams. Finally, we present an experimental confirmation of the polarization structure of Hermite-Gaussian laser beams.

We begin by summarizing the properties of the solutions of the paraxial scalar wave equation in Sec. II. We then construct paraxial beam solutions of Maxwell's equations from the solutions of the paraxial scalar wave equation in Sec. III. Polarization properties of Hermite-Gaussian laser beams are then discussed in Sec. IV. Finally, in Sec. V, we present an experimental confirmation of the polarization properties Hermite-Gaussain laser beams.

II. THE SCALAR WAVE EQUATION

The scalar wave function $\psi(\vec{r},t)$ in free space satisfies the source free wave equation

$$\left[\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\psi(\vec{r},t)=0, \tag{1}$$

where ∇^2 is the three dimensional Laplacian operator and c is the wave speed. For electromagnetic waves c is the speed of light. For quasimonochromatic waves of angular frequency ω, propagating predominantly in the z direction, the wave amplitude has the form

$$\psi(\vec{r},t)=\psi(\vec{r})e^{i(kz-\omega t)}, \tag{2}$$

where $\psi(\vec{r})$ describes the variation of the wave amplitude in transverse directions (x-y plane). Propagation constant k is related to the wavelength λ and the angular frequency ω of the wave by

$$k=\frac{\omega}{c}=\frac{2\pi}{\lambda}. \tag{3}$$

For paraxial beams, the energy is concentrated near the axis of the beam and the transverse profile of the beam, as described by $\psi(\vec{r})$, varies little with z over propagation

*Present address: U.S. Air Force Academy, Colorado Springs, CO 80840.

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Second-harmonic generation and photon bunching in multimode laser beams

Yujiang Qu and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701
(Received 10 August 1992)

Second-harmonic generation in multimode laser beams is studied experimentally. A multimode laser beam derived from a laser oscillating simultaneously in many longitudinal modes that have the same transverse profile (TEM_{00}) is focused into a nonlinear crystal. Noncritical phase matching is utilized to generate the second harmonic. The number of longitudinal modes is systematically varied from 1 to 7 and the dependence of the efficiency of second-harmonic generation on the number of modes in the fundamental beam is studied. Experimental results clearly demonstrate the increasing degree of photon bunching in the fundamental beam as the number of modes increases. Experimental results are discussed and compared with the predictions of theoretical models.

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Optical harmonic generation by laser beams in nonlinear-optical materials has been studied for a long time. The simplest example of harmonic generation is second-harmonic generation (SHG). Many aspects of second-harmonic generation have been investigated in the literature [1]. In particular, the dependence of the SHG on the statistical properties of the fundamental light beam has been investigated both experimentally [2–4] and theoretically [4,6]. Second-harmonic generation is an example of two-photon processes. Recent experiments on two-photon absorption and second-harmonic generation [3,5] have demonstrated that the second-order intensity correlations of the fundamental beam are directly reflected in the two-photon signal and that the intensity correlations of a fundamental beam can be measured in harmonic generation and multiphoton experiments [3,7–10].

In a multimode laser, operating far above threshold, photon statistics are no longer Poissonian even when the modes are noninteracting. As the number of modes increases the underlying field statistics become Gaussian. This may be considered a consequence of the central limit theorem. The associated photon sequence will be super-Poissonian and exhibit bunching. This means that the efficiency of SHG will depend on the number of modes in the fundamental beam and we can expect an enhancement of SHG efficiency as the number of modes increases. Early SHG experiments with multimode laser beams gave some indications of this enhancement [6,8–10]. These studies, however, did not provide a systematic and quantitative test of the theoretical predictions as the number of longitudinal modes in the fundamental beam was not controlled. They also did not consider the effects of laser gain profile on the distribution of power among different modes. In this paper, we report experimental investigations of SHG with multimode fundamental beams. The number of longitudinal modes in the laser was varied systematically from 1 to 7 and the dependence of the second-harmonic signal on the number of modes in the fundamental beam was studied. We also take into account the transverse spatial structure of the fundamental beam and the variations of the mode intensities due to the gain profile of the laser medium. In all experiments the total fundamental power was held constant at the same value. This allows a direct comparison of the efficiency of second-harmonic generation as the number of modes in the fundamental beam is varied.

We begin by summarizing the theoretical results. This is followed by a description of the experimental setup and a comparison of the results of the experiment with the theoretical predictions.

THEORY

The electric field of a linearly polarized laser beam containing N longitudinal modes and propagating in the z direction can be written as

$$\mathcal{E}_f(\mathbf{r},t)=\sum_{m=1}^{N} E_{fm}(\mathbf{r})e^{i(k_m z-\omega_m t)} , \tag{1}$$

Mode frequencies ω_m can be expressed in terms of the average frequency $\omega_0=\sum_m \omega_m/N$ of the fundamental beam and the mode frequency separation $\Delta\omega$ as

$$\omega_m=\omega_0-(N-2m+1)\Delta\omega/2 . \tag{2}$$

Wave number k_m for the mth mode is given by

$$k_m=n_f\omega_m/c , \tag{3}$$

where n_f is the refractive index of the medium for the fundamental wave. The amplitude $E_{fm}(\mathbf{r})$ of the mth mode satisfies the paraxial wave equation and describes the transverse profile of the beam. We neglect the variation of the transverse beam profile over the range of frequencies contained in the fundamental beam. This means that all modes are assumed to have the same transverse spatial structure. The multimode field of Eq. (1) propagates inside a nonlinear medium and induces a nonlinear polarization proportional to the square of the fundamental field amplitude. This polarization acts as the source for the second-harmonic fields. If we write the second-harmonic field at frequency ω_s as

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Resonance fluorescence with squeezed-light excitation

Reeta Vyas and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701
(Received 30 August 1991)

Resonance fluorescence from a single two-level atom driven by a beam of squeezed light is studied in the weak-field limit. We consider the situation where the atom is coupled to the ordinary vacuum and only a few field modes corresponding to the driving field are squeezed. The field produced by the degenerate optical parametric oscillator is used as the driving field. Heisenberg equations of motion are solved in the steady-state and analytic expressions for the fluorescent-light intensity and the spectrum of fluorescent light are derived. We also consider photon statistics of fluorescent light. In particular, squeezing, antibunching, and sub-Poissonian statistics of fluorescent photons are discussed, and analytic expressions for the quadrature variance and the two-time intensity correlation function are presented. Contrary to the case of coherent excitation, the second-order intensity correlation function does not factorize. This and other differences are discussed, and curves are presented to illustrate the behavior of various quantities. We also present results for thermal excitation of the atom.

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I. INTRODUCTION

A single two-level atom interacting with an electromagnetic field is a fundamental model of quantum mechanics. This simple dissipative quantum system lies at the heart of the physics of light-atom interaction. An interesting aspect of this problem is the phenomenon of resonance fluorescence when the atom is illuminated by a light beam whose frequency is nearly resonant with the transition frequency of the atom. The fluorescent light under these conditions displays many purely quantum-mechanical features [1]. These features are most clearly reflected in the photon statistics of fluorescent light [2,3]. The problem of photon statistics in resonance fluorescence has been treated by a number of workers using several different techniques [4]. Most treatments assume the incident light beam to be in a coherent state. A complete solution to photon statistics in resonance fluorescence under coherent illumination has been given recently [5]. Other models of the incident light beam that take into account the fluctuations of incident light have also been considered. These include the phase-diffusion model [6], the chaotic-field model [7], and the jump models [8] of phase and amplitude fluctuations. More recently, nonclassical states of the driving field have also been considered. Gardiner considered the problem of radiative decay in the presence of broadband (white-noise) squeezed light [9]. The possibility of a subnatural linewidth in resonance fluorescence under similar conditions has been investigated by Carmichael, Lane, and Walls [10]. More realistic models of squeezed light, where only a few modes are squeezed (colored squeezed light), have been considered by Ritsch and Zoller in the discussion of the atomic absorption spectrum [11]. In this paper we discuss the interaction of a single two-level atom with a finite-bandwidth squeezed light. The model of squeezed light that we adopt corresponds to light from a degenerate parametric oscillator operating below threshold [12,13]. This field can be modeled by two real Gaussian processes with different variances and correlation times [14]. Because of the finite correlation time of the incident field, atomic states and field states develop correlations during their dynamical evolution. Thus, unlike the case of coherent excitation, where the factorization of correlation functions [2,3,5,15] leads to a simplified description, the problem of photon statistics becomes complex in the present case. Nevertheless, for weak driving fields appropriate to subthreshold degenerate parametric oscillators, it is still possible to gain some insight into the behavior of a two-level atom when the exciting field is squeezed.

In Sec. II we derive the equations of motion governing the time evolution of atomic and field operators. Solutions to these equations are used to discuss the time evolution of fluorescent-light intensity in Sec. III. The spectrum of scattered light is calculated in Sec. IV. Photon statistics and the two-time intensity correlation function of scattered light are discussed in Sec. V, and Sec. VI presents results for thermal beam excitation of the atom. Finally, the principal results of this paper are summarized in Sec. VII.

II. EQUATIONS OF MOTION

Consider a two-level atom having energy states $|1\rangle$ and $|2\rangle$ separated by an energy gap $\hbar\omega_0$ and interacting with an electromagnetic field via a dipole interaction. The Hamiltonian for this system, in the rotating-wave approximation, is

$$\hat{H}=\hbar\omega_0\hat{R}_3+i\omega_0\boldsymbol{\mu}\cdot[\hat{\mathcal{A}}^{(-)}(0,t)\hat{\sigma}_-(t)-\hat{\mathcal{A}}^{(+)}(0,t)\hat{\sigma}_+(t)] +\hat{H}_F \, . \tag{1}$$

Here $\hat{H}_F$ represents the energy of the electromagnetic field. $\hat{R}_1(t)$, $\hat{R}_2(t)$, and $\hat{R}_3(t)$ are three dynamical spin

Enhancement of photon antibunching by passive interferometry

Y. Qu, Min Xiao, G. S. Holliday,* and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701

H. J. Kimble
Norman Bridge Laboratory of Physics 12-33, California Institute of Technology, Pasadena, California 91125
(Received 23 September 1991)

Photon antibunching in intracavity second-harmonic generation and intracavity atomic absorption inside a high-Q optical cavity is discussed. A linearized description of quantum dynamics in the weak-field limit is used because the system size, as characterized by the number of photons n_0 needed to probe the nonlinearity of the system, is often quite large for realistic parameters. In both cases standard results are recalled that demonstrate that antibunching is a small effect given by $g^{(2)}(0)-1\approx -1/n_0$ for large values of n_0. We show that when higher-order terms that are usually ignored in linearized treatments are retained, $g^{(2)}(0)$ satisfies the lower bound $g^{(2)}(0)\geq 0$ even for small values of n_0. It is further shown that by employing a high-Q cavity to suppress the coherent part of the spectrum of field fluctuations, perfect antibunching can be achieved even for large systems. Conditions under which this is possible are derived and curves are presented to illustrate the behavior.

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I. INTRODUCTION

Many nonlinear dissipative systems in optical physics involving, for example, the atom-field interaction or the parametric interaction between different modes of a cavity exhibit nonclassical features [1–6]. These features are conveniently described in terms of the statistical properties of the electromagnetic field. Unfortunately, most of these systems are such that many photons are needed to explore the nonlinearity of the system so that system dynamics are such that a very small quantum noise evolves around a classical steady state. The smallness of quantum noise makes a linearized treatment of the quantum dynamics of the system possible, with quantum effects inherently small compared to the size of the classical steady state. Examples of case in point are photon antibunching effects predicted in second-harmonic generation [1,3] and multi atom optical bistability [4–6]. In both cases the antibunching effect is inversely proportional to the size of the system as characterized by some parameter n_0. For multiatom bistability in the good-cavity limit this parameter is the saturation photon number, and for intracavity second-harmonic generation it is the threshold photon number. In principle, at least, it is possible to enhance the relative size of quantum effects by reducing the system size. In optical bistability, for example, the system-size parameter n_0 (the saturation photon number) was of the order of 10^3 for the work of Ref. [7]. The corresponding antibunching effect is then predicted to be considerably less than 1%. By searching for appropriate atomic species (Cs, for example) and working with short (~ 1 mm in length) optical cavities of small mode volume, the saturation photon number can be reduced down to below unity. By further utilizing a high-finesse ($\mathcal{F}\geq 10^4$) cavity and employing an atomic cooperativity parameter $C\sim 20$, the antibunching effect can be increased [6] to about 20% as has been observed in recent experiments [17]. The corresponding problem of reducing the system size in second-harmonic generation is more formidable [3]. For most experimental situations the threshold photon number is of the order of 10^6–10^{-8}. The corresponding antibunching effect is of the order of 10^{-6}–10^{-8}. By using high-finesse optical cavities and crystals with a large nonlinear coefficient, the threshold photon number can perhaps be lowered to 10^3-10^4. The resulting antibunching effect (of order 10^{-3}–10^{-4}) is still woefully small. Thus the approach of system-size reduction is not always feasible. Furthermore, with a reduction in system size the simplified linearized treatment of quantum dynamics may be rendered invalid. From the experimental point of view then the general problem we face is this: how do we enhance and therefore measure a small quantum effect riding on a strong classical (coherent) background. One approach to this problem, based on an interference experiment, was discussed by Bandilla and Ritze [8]. Here we explore yet another approach [9], which employs a passive filter cavity, external to the system producing quantum effects, to provide a variable for the classical (coherent) carrier while leaving the spectrum of quantum fluctuations largely intact, thereby enhancing otherwise small quantum effects. The quantum effects of interest to us throughout this paper are photon antibunching and sub-Poissonian photon statistics [3,5]. The fields considered in this paper also exhibit squeezing. In general, however, squeezing and antibunching refer to different nonclassical properties of the electromagnetic field. We are not concerned here with squeezing of the fields, although the method of this paper can also be applied to a discussion of squeezing. In Sec. II we present a general approach to the problem of filtering by a high-Q optical cavity. Section III applies these results to antibunching and sub-Poissonian photon

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Photon correlation effects in second harmonic generation

Yujiang Qu and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, AR 72701, USA

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Quantitative measurements of the dependence of second harmonic signal on the photon statistical properties of the fundamental beam are reported. Using the light from a single-mode laser operating near threshold it is shown that the second harmonic signal directly measures the degree of two-photon bunching in the fundamental beam. In particular, thermal light is shown to be twice as effective in generating second harmonic signal as coherent light for the same intensity.

Multiphoton processes are very sensitive to the [illegible] statistical properties of the incident field since [illegible] probe higher order correlation functions of the [illegible] [1–4]. The case of two-photon processes has [illegible] investigated by several workers [2,3]. A special [illegible] of two-photon processes is second harmonic [illegible] (SHG) in which two photons from a fun[illegible] beam of frequency ω combine to generate [illegible] photon at the second harmonic frequency [illegible] a nonlinear crystal [4]. The second har[illegible] (subscript s) intensity (photons/second) is [illegible] to the fundamental (subscript f) light inten[illegible]

$$[illegible] K\langle :\hat{I}_{\mathrm{f}}^2(t): \rangle \,, \tag{1}$$

[illegible] brackets denote averaging with respect to [illegible] of the field, colons denote normal ordering [illegible] ordering. The constant K for a fundamental [illegible] cylindrically symmetric gaussian spatial [illegible] takes the form [5]

$$[illegible] \left(\frac{64\pi^4\hbar d^2}{n_f^2\lambda_f^4}\right) l h_{\mathrm{m}}(B,\xi) \,. \tag{2}$$

[illegible] the dielectric permittivity of the free space, [illegible] refractive index of the crystal at the fun[illegible] frequency, λ_{f} is the fundamental wave[illegible] space, d is the nonlinear coefficient (m/[illegible] the crystal length. The dimensionless [illegible] (B,ξ) takes into account diffraction, fo[illegible] the degree of phase matching [5]. The double refraction parameter B (dimensionless) depends on the beam walkoff angle ρ, and the focusing parameter $\xi = l/b$ measures the strength of focusing where b is the confocal parameter of the fundamental beam. For noncritical phase matching, when both the fundamental and the second harmonic beams propagate normal to the optic axis (90° phase matching), $B=0$ and $h_{\mathrm{m}}(0,\xi)$ has its maximum value 1.07 for $\xi=2.84$.

Equation (1) shows that the second harmonic signal is proportional to the second order intensity correlation function of the fundamental beam. For a coherent light beam this correlation function factorizes so that $\langle \hat{I} \rangle_{\mathrm{coherent}} = K\langle \hat{I}_{\mathrm{f}} \rangle^2$. We can then introduce a dimensionless statistical efficiency $\eta_{\mathrm{s}} \equiv \langle \hat{I}_{\mathrm{s}} \rangle / \langle \hat{I}_{\mathrm{s}} \rangle_{\mathrm{coherent}}$ of SHG, which from eq. (1) leads to

$$\eta_{\mathrm{s}} \equiv \langle \hat{I}_{\mathrm{s}} \rangle / K \langle \hat{I}_{\mathrm{f}} \rangle^2 = g_{\mathrm{f}}^{(2)}(0) \,, \tag{3}$$

where the normalized second order intensity correlation function of the fundamental beam is given by

$$g_{\mathrm{f}}^{(2)}(\tau) = \langle : \hat{I}_{\mathrm{f}}(t)\, \hat{I}_{\mathrm{f}}(t+\tau) : \rangle / \langle \hat{I}_{\mathrm{f}} \rangle^2 \,. \tag{4}$$

Statistical efficiency η_{s} unlike the ordinary efficiency of SHG, is independent of the absolute intensity of the fundamental beam. It is certainly true that if the fundamental beam is attenuated by a linear filter it does not change η_{s}. It is a more appropriate measure for comparing the performance of different states of the field in SHG as different states of the field are often available with widely differing intensities. For

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Intensity correlation functions of the laser with multiplicative white noise

K. J. Phillips, M. R. Young, and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701
(Received 11 January 1991)

Two-time intensity correlations in a laser with multiplicative white noise have been studied experimentally. Multiplicative noise was introduced by applying a Gaussian-noise voltage to an intracavity acousto-optic modulator in a He:Ne laser operating near threshold. We observe both a shift and a broadening of the peak in the correlation-time versus mean-light-intensity curve. These observations are in agreement with the previous measurements of the steady-state properties of the laser near threshold in the presence of multiplicative noise.

I. INTRODUCTION

The primary source of noise in lasers is spontaneous emission. This noise together with the nonlinearity of light-matter interaction determines the fluctuation properties of laser light [1]. In practice, however, other sources of noise also contribute to fluctuations of laser light. Since lasers are open systems coupled to their environment, random fluctuations of the environment also influence the statistical properties of laser light. These fluctuations of the environment enter the laser via pump or loss fluctuations. Noise due to these fluctuations is proportional to the electric-field amplitude in the lowest-order approximation. This type of noise is called multiplicative noise [2]. It is also referred to as parametric noise or external noise.

Lasers are but one example of open systems that are subject to multiplicative noise. The role of multiplicative noise in such systems has been of great interest recently [2,3]. The presence of multiplicative noise induces qualitatively new phenomena that cannot be induced by additive noise, white or colored, alone. It may cause the appearance of new statistically favored states. It may cause the shift of oscillation threshold. Many of these predictions have been confirmed experimentally in a variety of systems [4–7]. In a He:Ne laser, for example, the effects of multiplicative white noise have been investigated systematically in photoelectric counting experiments [5]. These experiments confirm the predicted shift of laser threshold and enhanced intensity fluctuations in the presence of multiplicative noise. The importance of multiplicative noise in lasers was realized when the statistical properties of dye lasers were investigated experimentally [7]. Experiments revealed that the observed intensity fluctuations could be accounted for only by including multiplicative noise, in addition to the additive spontaneous emission noise, in the equation of motion for the laser field amplitude [8]. Early theoretical work on dye lasers had predicted that the complex structure of dye molecules may alter the nature of nonlinearity [9]. In particular, the presence of triplet state absorption in the same spectral region as the singlet laser emission may induce a first-order phase-transition-like behavior in dye-laser fluctuations. This has, however, not been observed. It is not clear what role, if any, triplet absorption does play. Experiments that have been carried out to date seem to suggest that triplet states do not play an essential role. This conclusion can only be tentative since many of the effects induced by multiplicative noise are similar to those that would be induced by triplet absorption [9]. Effects of multiplicative noise and triplet absorption could be delineated in experiments that study the role of multiplicative noise in a systematic way by varying the strength of multiplicative noise in a controlled fashion. Such experiments have not been carried out in dye lasers. In simple systems, such as a He:Ne laser, systematic studies of the effects of multiplicative noise have been carried out by Young and Singh [5]. In these experiments the steady-state intensity fluctuations were studied in photoelectric counting experiments. Corresponding experimental studies of the effects of multiplicative noise on the correlation functions of laser light have not yet been reported although this problem has been treated theoretically by several workers [6,10,11]. In this paper we wish to describe results of photoelectric correlation experiments carried out on a He:Ne laser where multiplicative white noise was introduced in controlled amounts by applying a Gaussian-noise voltage to an intracavity acousto-optic modulator. The paper is organized as follows. In Sec. II we briefly review the steady-state fluctuation properties of a laser with multiplicative noise. We include both quantum noise due to spontaneous emission and multiplicative noise due to loss fluctuations. Section III discusses steady-state correlation functions of the laser. Experimental setup and procedures are described in Sec. IV. Finally, experimental results and principal conclusions are presented in Sec. V. Although both gain and loss fluctuations may contribute to multiplicative noise in lasers we confine ourselves to the case where loss fluctuations dominate. In the first approximation, especially when external fluctuations are small, both gain and loss fluctuations lead, qualitatively, to similar results.

II. EQUATION OF MOTION

Consider a single-mode laser operating close to threshold whose losses are modulated by a white-noise Gauss-

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NON-EQUILIBRIUM TRICRITICAL BEHAVIOR IN LASERS

MANSOUR MORTAZAVI and SURENDRA SINGH
Department of Physics, University of Arkansas, Fayetteville, AR 72701, USA

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Tricritical behavior in lasers is reviewed briefly. Threshold behavior in the laser with a saturable absorber is interpreted in terms of first- and second-order phase transitions within the framework of Landau theory (mean field). The intersection of the lines of first- and second order phase transitions in this system provides an example of a non-equilibrium tricritical point. By varying the operating parameters of a He:Ne laser with an intracavity Ne absorber it was possible to operate the laser near the tricritical point. Experimental results are compared with the predictions of mean field theory.

1. Introduction

Many physical systems operating far from thermodynamic equilibrium exhibit transitions from one state of order to a different one when certain parameters of these systems are varied. These non-equilibrium transitions exhibit remarkable similarities to phase transitions in systems that are in thermodynamic equilibrium. These analogies are based on the concepts and results of classical Landau theory and not those of the modern renormalization group theory. The concept of phase transitions in non-equilibrium systems has proved extremely useful because it illustrates that rather complex systems drawn from diverse fields can be understood and described in terms of a few simple concepts.[1] Although the examples of non-equilibrium phase transitions can be found in many systems in physical, biological and even social sciences, none has been more successfully modelled than that associated with the laser threshold. The appearance of a coherent field mode with a well defined phase from an ensemble of thermal field modes with random phase suggests Bose-Einstein condensation; the breaking of phase symmetry is characteristic of a second-order phase transition. Based on these similarities detailed formal phase transition analogies were [illegible] by several authors.[2–3] It was possible to identify variables of the laser that are analogs of the order parameter, the free energy and the entropy of a system undergoing second order phase transition. Laser systems are capable of exhibiting further classes of instabilities that resemble first-order phase transitions. This was first revealed in the theoretical treatment of the laser with a saturable absorber (LSA).[4] The non-equilibrium first-order phase transition in the LSA was

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Photon-counting statistics of the degenerate optical parametric oscillator

Reeta Vyas and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701
(Received 22 May 1989)

Nonclassical light beams generated by the degenerate optical parametric oscillator operating below threshold are analyzed in terms of photoelectron-counting sequences. The positive-P representation is used to calculate the generating function for photoelectron statistics in a closed form. This generating function is used to derive expressions for the photoelectron-counting and waiting-time distributions. The dependence of these distributions on mean photon number inside the cavity and efficiency of detection is studied. The relationship between photoelectron-counting sequence and the photon emission sequence is used to present a simple physical picture of light beams produced by the degenerate parametric oscillator.

I. INTRODUCTION

Squeezed states of light have been observed in a variety of physical systems.[1-3] These states do not admit a positive nonsingular diagonal representation in terms of coherent states and are, therefore, an example of nonclassical states of the electromagnetic field. Since squeezing only refers to the variance of the two quadrature components of the electric field, it does not fully characterize these states. With experimental realization of these states, increasing attention is being paid to their quantum-statistical properties.[4,5] These properties for idealized squeezed states are well known.[6-9] The systems in which squeezed states have been observed experimentally are dissipative nonlinear systems, and photon statistical properties of squeezed states produced by these systems have received much less attention.

The largest amount of squeezing has been observed in an optical parametric oscillator (OPO) operating below threshold.[2,3] This simple dissipative quantum system has played an important role in recent studies of squeezing. In an OPO (Ref. 10) a strong pump beam interacts with a nonlinear crystal and is frequency down-converted into two beams of smaller frequencies inside an optical cavity. If the two beams produced in down conversion have the same frequency, then the oscillator is termed a degenerate parametric oscillator (DPO); otherwise it is termed a nondegenerate parametric oscillator (NDPO). A quantum-mechanical treatment of the OPO is of course essential since it generates light with nonclassical properties.

For an oscillator a distinction must be made between intracavity photon statistics and the statistics of photons emitted by the cavity. Intracavity statistics are not directly observable. The statistics of photons emitted by the cavity can be measured in photon-counting experiments. The statistics of the field inside and outside the cavity are, of course, related. Many recent studies of the quantum-statistical properties of the DPO have centered around the calculation of the spectrum of squeezing[11,12] inside and outside the cavity because of the subtleties involved in the detection of squeezed light. Intracavity field statistics were discussed by Drummond, McNeil, and Walls[13] by using the complex-P representation and by Graham by using the Wigner function[13]. More recently, Wolinsky and Carmichael[4,14] have provided a complete description of the quantum-statistical properties of the intracavity field by using the positive-P representation. For the photons escaping the cavity, the mean and variance of photon counts have also been calculated by Collett and Loudon.[15]

In this paper we discuss the quantum-statistical properties of photon beams generated by an OPO as measured by a detector placed outside the cavity. These properties can be studied in photoelectric-counting and correlation experiments with low-intensity light beams appropriate for an OPO below threshold. From the measured photoelectron statistics, photon statistics of the incident light beam can be derived. For a detector of unit efficiency each photodetection corresponds to an emission of a photon by the cavity. In this case, the photoelectric-counting sequence and the photon emission sequence are equivalent. We begin by expressing the photoelectron-counting statistics in terms of a generating function in Sec. II. The statistics of the waiting time between successive photoelectric counts can also be derived from the same generating function. In Sec. III the c number equations of motion for the DPO operating below threshold are presented. This is done by using the positive-P representation. The solutions to these c-number equations are used to obtain a closed form expression for the generating function. From this generating function exact expressions for the photoelectron-counting distribution and the waiting-time distribution are derived in Sec. IV. Intracavity photon statistics are discussed in Sec. V. We conclude by summarizing the principal results of the paper in Sec. VI.

II. THE GENERATING FUNCTION

Consider a photoelectric detector illuminated by a stationary weak beam of light. The probability $p(m,T)$ of detecting m photoelectric counts at the output of the detector in a time interval T is given by[16]

PHYSICAL REVIEW A VOLUME 39, NUMBER 3 FEBRUARY 1, 1989

Photoelectron waiting times and atomic state reduction in resonance fluorescence

H. J. Carmichael, Surendra Singh, Reeta Vyas, and P. R. Rice*
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701
(Received 12 September 1988)

Photoelectron counting sequences for single-atom resonance fluorescence are studied. The distribution of waiting times between photoelectric counts is calculated, and its dependence on driving-field intensity and detection efficiency is discussed. The photoelectron-counting distribution is derived from the waiting-time distribution. The relationship between photoelectron counting sequences and photon emission sequences is discussed and used to obtain an expression for the reduced state of the atom during the waiting times between photoelectric counts. The roles of irreversibility and the observer in atomic state reduction are delineated.

I. INTRODUCTION

The fluorescent photons emitted by a single coherently driven two-level atom exhibit the nonclassical property of photon antibunching.[1-3] The antibunching of fluorescent photons is seen in temporal correlations between photoelectric counts; the detection of one photon makes the detection of a second, after just a short delay, improbable. Photon antibunching is traditionally defined in terms of the degree of second-order temporal coherence $g^{(2)}(t,t+\tau)$. This is the joint probability for recording photoelectric counts in the intervals $[t,t+\Delta t)$ and $[t+\tau,t+\tau+\Delta t)$, normalized by the probability for two independent photoelectric counts. For antibunched light the joint probability for recording photoelectric counts closely spaced in time falls below the probability for statistically independent counts (separated by a time longer than the coherence time); thus, $g^{(2)}(t,t)<1$.

The antibunching of fluorescent photons is also reflected in the sub-Poissonian character of the probability density $p(n,t,t+T)$ for recording n photoelectric counts in the interval $[t,t+T)$.[4] $p(n,t,t+T)$ can be derived from $g^{(2)}(t,t+\tau)$, although the detailed algebraic relationship is quite complicated. Both $g^{(2)}(t,t+\tau)$ and $p(n,t,t+T)$ have been calculated for single-atom resonance fluorescence by a number of workers.[1-10] Because of the complexity of general expressions in the time domain, some workers only give the Laplace transform for the photoelectron counting distribution, or give explicit time-dependent expressions only for limiting cases, such as short and long counting times.

Recent theoretical work on "quantum jumps"[11,12] has drawn attention to the distribution of waiting times between photon emissions as another useful quantity for characterizing photon statistics—in terms of measured quantities, the distribution of waiting times between photoelectrons. By "waiting time" we mean the time τ between a photoelectric count recorded at time t, and the next, recorded at time $t+\tau$. If photoelectron sequences can be described by a Markov birth process, a single conditional probability density $w(\tau|t)$ specifies the distribution of waiting times between every pair of photoelectrons. We call this the photoelectron waiting-time distribution. Photoelectron waiting times for coherent light are exponentially distributed.[13] Antibunching implies that photons tend to be separated in time. The distribution of waiting times should then tend to peak around the average time between photoelectric counts.

Photoelectron waiting times are certainly not new to the field of photon statistics. Indeed, when a time-to-amplitude converter is used for a delayed coincidence measurement, the raw data provide the distribution of waiting times between photoelectric counts. However, when the count rate is sufficiently low, this distribution is proportional (aside from dead-time corrections) to $g^{(2)}(t,t+\tau)$.[14] This relationship provides the technique used to measure $g^{(2)}(t,t+\tau)$ in the experiments of Kimble *et al.*[3] on photon antibunching in resonance fluorescence. Thus, the waiting-time distribution and its relationship to $g^{(2)}(t,t+\tau)$ are known. But the waiting-time distribution has not been mentioned until recently[12,15,16] in the large theoretical literature on resonance fluorescence. This is a deficiency, since it provides a clearer physical picture of photon emission sequences, corresponding photoelectron counting sequences, and their nonclassical properties, than $g^{(2)}(t,t+\tau)$. In this paper we revisit the problem of single-atom resonance fluorescence and focus attention on the waiting-time distribution. (We will discuss waiting times between photon emissions as well as between photoelectrons. When the distinction is not important we simply refer to "the waiting times" or "the waiting-time distribution.")

There are probably two main reasons for the lack of attention paid to $w(\tau|t)$ in early work on resonance fluorescence. The first is that $g^{(2)}(t,t+\tau)$, not $w(\tau|t)$, is the quantity accessible to measurement. It might be asked, why not use a time-to-amplitude converter to measure the quantity it gives directly, the photoelectron waiting-time distribution $w(\tau|t)$? The problem is that photoelectric detection is very inefficient. The average time between photoelectric counts is unavoidably much longer than the correlation time of the fluorescent light. Then $w(\tau|t)$ is proportional to $g^{(2)}(t,t+\tau)$; $w(\tau|t)$ can be measured, but only when it effectively reduces to $g^{(2)}(t,t+\tau)$. Actually, the proportionality between these quantities does not hold for all τ, but it holds over many correlation

Waiting-time distributions in the photodetection of squeezed light

Reeta Vyas and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701
(Received 24 March 1988)

Distribution of waiting-time intervals between the arrivals of successive photons on a photocathode illuminated by a beam of light is discussed. Analytic expressions for the conditional and unconditional distributions for squeezed light are derived in the high degeneracy limit. Results for binomial and thermocoherent states are also given. Curves are presented to illustrate the behavior.

I. INTRODUCTION

Squeezed states of light have been observed in a number of experiments.[1-4] Similar to the states exhibiting photon antibunching,[5] squeezed states[6] are nonclassical states. For these states variance in one of the quadrature components of the electric field is smaller than the corresponding variance for a coherent state. The nonclassical nature of these states is also reflected in the fact that the corresponding phase space density in the coherent state diagonal representation[7,8] does not exist as an ordinary probability density. Photon number distributions for squeezed states have been discussed by a number of authors[9-14] and they reveal several interesting features of these states. Depending on the parameters characterizing a squeezed state, such a state may exhibit sub-Poissonian or super-Poissonian photon statistics.[14] Since, for short times at least, sub-Poissonian (super-Poissonian) photon statistics reflect antibunching (bunching),[15] squeezed states are capable of exhibiting antibunching or bunching under suitable circumstances. The properties of photon antibunching or bunching are best visualized in terms of the theory of photoelectric detection.[16-18] Consider a photoelectric detector illuminated by a beam of light. For an ideal photodetector of unit detection efficiency and zero dead time, the photoelectric pulses appearing at the output of the photodetector are in one-to-one correspondence with the arrival of photons on the photocathode. In terms of this sequence of photoelectric pulses, photon antibunching implies that the detection of a photon at a certain time t renders the detection of another photon immediately following the first less probable. The opposite is implied by photon bunching. This means that for an antibunched beam of light, successive photoelectric pulses will be separated, on the average, by large time intervals. We may consider this to be a reflection of the tendency of the photons in the light beam to be separated in time. This physically appealing picture of the behavior of photons in a light beam emerges from photodetection theory. Photons themselves do not lend directly to such an interpretation. This time-dependent behavior of photons will be reflected in the distribution of the time interval between successive photodetections.[18,19] It is the purpose of this investigation to derive the time interval distribution for photons in a squeezed state.

In Sec. II we present an outline of the photodetection theory leading to a general expression for the time interval distribution. This expression is used to discuss the behavior of photons in a squeezed beam in Sec. III. We also discuss the behavior of waiting time distributions for binomial and thermocoherent states of the field in Sec. IV.

II. TIME INTERVAL DISTRIBUTION FUNCTION

In order to discuss the time interval distribution for a light beam we need the threefold joint probability of photodetection, for the time interval T between successive photodetections is defined in terms of three events: a photodetection event at some time t_1, no photodetection events in the interval $[t_1,t_2(=t_1+T)]$, and one photodetection at time t_2. First let us recall that the probability of detecting n photoelectric events in $[t_1,t_2]$ is[17]

$$p(n,t_1,t_2)=\left\langle :\frac{\hat{U}^n}{n!}e^{-\hat{U}}:\right\rangle , \tag{1}$$

where

$$\hat{U}=\eta\int_{t_1}^{t_2}\hat{I}(t)dt , \tag{2}$$

and $\hat{I}(t)$ is the photon flux operator (number of photons per unit time) and $:\cdots:$ denotes the time-ordered normal product of operators. The detection efficiency η depends on the characteristics of the detector. The angular brackets denote averaging with respect to the state of the field. From Eqs. (1) and (2) the probability of detecting one photon in the interval $[t-\Delta t,t]$ is found to be

$$p(1,t-\Delta t,t)=\eta\langle\hat{I}(t)\rangle\Delta t , \tag{3}$$

where Δt is some small time interval. We also find from Eq. (1) that the probability that no photodetection occurs in the interval $[t_1,t_2]$ is

$$G(t_1,t_2)=\left\langle :\exp\left[-\int_{t_1}^{t_2}\hat{I}(t)dt\right]:\right\rangle . \tag{4}$$

Equations (3) and (4) will be used later in the paper. The single-fold photoelectric counting formula (1) is well known.[16-18] The threefold photoelectric counting formula ($t_0<t_1<t_2<t_3$)

PHYSICAL REVIEW A VOLUME 38, NUMBER 1 JULY 1, 1988

Effects of multiplicative white noise on laser light fluctuations

M. R. Young and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701
(Received 20 July 1987; revised manuscript received 26 February 1988)

The steady-state statistical properties of the single-mode laser with multiplicative white-noise loss fluctuations are investigated. Analytic expressions for the intensity distribution and its moments are derived and it is shown that the multiplicative noise significantly modifies the laser characteristics. These predictions are then tested by photoelectric measurements of a He:Ne laser with fluctuating loss. Good agreement between theory and experiment is obtained.

I. INTRODUCTION

Instabilities and phase transitions in nonequilibrium systems have been the subject of great interest in many branches of natural sciences.[1] In the neighborhood of an instability such systems are extremely sensitive to the presence of noise. There are two types of noise present in nonequilibrium systems.[2] The so-called additive noise arises due to the microscopic processes by which the system evolves and is independent of the macroscopic state of the system. Under the influence of additive noise a system executes a random walk in the state space leading to a distribution of the values of the state variables. This type of noise, however, leaves the local stability properties of the system unchanged. In particular, the critical points and the extrema of the probability distribution of the state variables coincide with the solutions of the deterministic system and no new instabilities which are not expected from a deterministic description arise. This type of noise has been studied extensively.

Multiplicative noise arises due to the randomness of the environment to which the system is coupled. This noise enters the system dynamics via its coupling to state variables. Although the importance of multiplicative noise in the context of electronic oscillators has been known for many years,[3,4] it is only recently that its importance for nonequilibrium systems has been realized. The effects of multiplicative noise are far less intuitive than those produced by additive noise. Thus, for example, multiplicative noise can change the local stability properties of the deterministic solutions. As a result the critical points as well as the extrema of the probability distribution of the state variables may differ from those of the deterministic description and new instabilities which are unexpected from the deterministic description may appear.[2-5] Since the multiplicative noise is state dependent it may not be negligible even for large systems. This is in contrast to additive noise which scales inversely as the system size and is usually important only in the neighborhood of an instability.

In quantum optics both types of noise have been studied in lasers where sources of both noise are present. The additive noise in the laser arises due to quantum-mechanical spontaneous-emission fluctuations and the multiplicative noise arises due to the fluctuations of the gain or the loss. Two laser systems that have been studied rather extensively are the single-mode He:Ne laser[6-9] and the single-mode dye laser.[10-13] The single-mode laser threshold instability in the He:Ne laser is perhaps the best studied nonequilibrium phase transition.[14] It has the phenomenology of a second-order phase transition and various experimental studies of this instability seem to indicate that, at least in the region of threshold, only the additive noise is important.[6-9] In the case of the dye laser, on the other hand, the experiments[10-13] suggest that the behavior of the dye laser is dominated by multiplicative noise[15] which arises due to pump fluctuations and the turbulence in the dye jet. It is not clear what the nature of the underlying instability is. Earlier theoretical investigations[16] had suggested a first-order-phase-transition-type instability. This has, however, never been observed. Furthermore, there are indications that the fluctuations of the dye flow are chaotic.[12] It remains to be investigated if the effects of chaotic fluctuations are any different from those of the random fluctuations. It is therefore clear that the fluctuations of the dye laser and the nature of the underlying instability as well as the role of multiplicative noise in lasers can be better understood by studying a system where the fundamental instability is well characterized and where multiplicative noise can be introduced in a controlled fashion. Also, since the additive spontaneous-emission noise is always present in lasers, it would be of interest to see what new features appear when additive and multiplicative noise strengths are similar.

In this paper[17] we report on the investigations of the effects of multiplicative noise on the single-mode laser threshold transition. Only the multiplicative white noise arising from laser loss fluctuations is considered here. In Sec. II we present an outline of the theoretical model which describe the effects of multiplicative noise and summarize the steady-state fluctuation properties predicted by the model. Section III describes the experimental setup and the method used to introduce multiplicative noise. A number of corrections to data and the procedure to determine certain key parameters are described in some detail. Experimental results and the principal conclusions of the paper are presented in Secs. IV and V, respectively.

Measurements of first-passage-time distributions in laser transients near threshold

M. R. Young* and Surendra Singh

Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701

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The first-passage-time (FPT) problem for the time evolution of the optical field produced by a Q-switched laser has been investigated theoretically and experimentally in the neighborhood of the laser threshold. Experiments were performed on a He–Ne laser operating near threshold in its steady state. Measurements for several operating points were made. Q switching was achieved with the help of an intracavity acousto-optic modulator. The measured FPT distributions are compared with the approximate analytic results for the FPT distribution and the numerical solutions of the laser Fokker–Planck equation based on the van der Pol oscillator model of the laser.

The threshold instability in the single-mode laser is an example of a nonequilibrium phase transition in which laser fluctuations undergo a smooth transition from a thermal state below threshold to a coherent state above threshold.[1] These fluctuations are a manifestation of the microscopic quantum noise that is due to spontaneous emission. Only fully quantum-mechanical treatments or semiclassical treatments based on Langevin equations that include quantum noise provide an adequate theoretical framework for describing the transition.[2–4] The predictions of these treatments have been tested and confirmed in many experiments[5–11] devoted to the study of the fluctuations in the laser. Some of the most sensitive tests of the theoretical predictions have come from the investigations of the transient characteristics of the laser performed by Arecchi and co-workers[7] and by Meltzer and Mandel.[9] These experiments investigated the variation of the photoelectric counting probability with time following the turn-on of the laser by measuring the light intensity at a fixed time after the laser was turned on. A complementary approach to this problem is to study the time that it takes the light intensity to reach a certain reference intensity after the laser is turned on. The time at which the first passage of the laser intensity through this reference value occurs is called the first-passage time (FPT).[12] This time is expected to fluctuate, reflecting the underlying quantum noise. The FPT measurements provide important new information on the role of noise in the dynamics of the laser. For example, if the quantum noise is important only during the early stages of laser dynamics, the FPT variance is expected to be flat as a function of the reference intensity.[13–15] This type of information is not available from intensity-versus-time measurements. If the noise is important throughout the entire evolution toward the steady state, no such behavior is to be expected.

It is easier to treat the transient FPT problem theoretically for a laser that operates far above threshold in its steady state. In this regime the concepts of the decay of an unstable state, scaling, and asymptotic approximation (low-noise limit) can be fruitfully employed to derive analytic results for the FPT distribution.[14,15] When the laser's steady state lies in the threshold region, the concept of the decay of an unstable state is not useful. The so-called states of the laser are not well defined because of the presence of relatively strong quantum noise. In this regime noise is important throughout the entire evolution, and it is more appropriate to consider the growth of the laser's electric field amplitude as a random walk under the influence of quantum noise. Direct evidence for this random walk was provided recently by means of the FPT measurements.[16] An analytic treatment of the FPT problem appears difficult to achieve in the threshold region. Arecchi and co-workers[13] have presented an approach based on the moments of the FPT distribution that can be expressed in terms of a series of nested integrals. In practice these expressions become increasingly difficult to evaluate as moments higher than the second are considered. Arecchi and co-workers tested the expressions for the first two moments by the FPT measurements of an electronic oscillator. FPT measurements in the scaling regime have been carried out by Roy and co-workers[17] on a dye laser operating far above threshold in the presence of strong external fluctuations. For a laser operating near threshold there have been no reported FPT measurements of its transient characteristics. In this paper we report on such measurements. Since the laser threshold is likened to a second-order phase transition,[1] these dynamical studies also provide an example of the critical dynamics of a nonequilibrium system.

We begin by outlining the theoretical approach to the FPT problem during the turn-on of a laser near threshold. We then discuss various analytic approximations to the FPT probability density and a method to compute it numerically. These results are then compared with the FPT measurements of the transients in a He–Ne laser operating near threshold. The laser was Q switched and was controlled to have its steady state in the region of threshold.

FIRST-PASSAGE-TIME PROBLEM

The scaled slowly varying complex electric field amplitude $E(t)$ of the single-mode laser obeys the following equation of motion[2–4]:

Statistical properties of a laser with multiplicative noise

M. R. Young and Surendra Singh

Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701

Received July 1, 1987; accepted October 13, 1987

Effects of multiplicative noise on the laser instability in a He–Ne laser are investigated by photon-counting measurements. Several new features, such as the shift of the threshold of laser oscillation and intensity fluctuations in excess of unity, are observed. Our measurements also reveal the importance of the nonlinearity of light–matter interaction even for negative pump-parameter values.

The role of parametric or multiplicative noise in nonequilibrium systems has been the subject of great interest in recent years.[1,2] On the experimental side, the dye laser seems to have become the prototype of nonequilibrium systems in which multiplicative noise plays an important role.[3–8] In the dye laser, however, the nature of the intrinsic instability, the role played by the triplet states[9] of the dye molecule, and the source and the nature[10] of multiplicative noise are not well characterized, although a stochastic theory with multiplicative noise that is due to pump fluctuations, in which the strength and the correlation time of the noise are treated as adjustable parameters, seems to give an adequate account of the statistical properties of the dye laser.[5,7] In order to understand the role of multiplicative noise, it would be desirable to perform experiments on a well-characterized system. We shall describe experiments carried out on a He–Ne laser, in which multiplicative noise was introduced by applying a noise voltage to an intracavity acousto-optic modulator (AOM). These measurements were made in a regime where multiplicative and additive spontaneous emission fluctuations have comparable strengths, and both must be taken into account.

An outline of the experimental setup is shown in Fig. 1. It consists of a single-mode He–Ne laser[11] operating near threshold. By application of a Gaussian noise voltage to an intracavity AOM, laser loss can be made to fluctuate. In the experiments described here, loss fluctuations were determined to have a flat spectrum up to frequencies in excess of 10 MHz. The corresponding correlation time, 10^{-7} sec, is 2–3 orders of magnitude smaller than the correlation time of the laser near threshold.[12] For small depths of modulation the transfer characteristics of the AOM were found to be linear. This means that the loss fluctuations may be treated as a Gaussian white-noise process.

In view of the preceding discussion the equation of motion for the scaled slowly varying complex field amplitude of the laser with loss fluctuations can be written as

$$\dot{E}(t) = E(t)(a - |E|^2) + q(t) + \xi(t)E(t), \qquad (1)$$

where $q(t)$ represents additive spontaneous emission fluctuations that are taken to be a complex Gaussian white-noise process with

$$\langle q(t)\rangle = 0, \qquad \langle q^*(t)q(t')\rangle = 4\delta(t - t'). \qquad (2)$$

Since, in the equation of motion, loss occurs multiplied by the field amplitude,[13] loss fluctuations give rise to the multiplicative noise represented by the last term in Eq. (1), which can be modeled by a Gaussian white-noise process, with

$$\langle \xi(t)\rangle = 0, \qquad \langle \xi^*(t)\xi(t')\rangle = 4Q\delta(t - t'), \qquad (3)$$

where Q is the strength of the multiplicative noise. In Eq. (1), a is the laser pump parameter. Equation (1) is interpreted as a Stratonovich stochastic differential equation[14] and is converted into a Fokker–Planck equation for the probability density of the field amplitude in the usual manner. From the steady-state solution of the Fokker–Planck equation we obtain the following expression for the intensity probability density[2,8]:

$$P_s(I) = N^{-1}(1 + QI)^{\nu-1}\exp[-(I/2Q)], \qquad (4)$$

where N is a normalization constant, $\nu = a/2Q + 1/2Q^2$, and $I = |E|^2$ is the light intensity. From Eq. (4), analytic expressions for the moments of the light in-

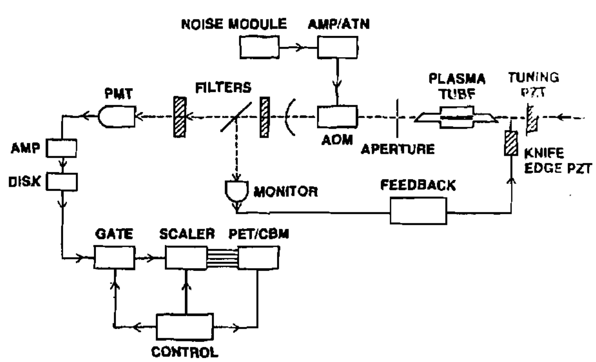

Fig. 1. An outline of the experimental apparatus. AMP/ATN, amplifier/attenuator; PET/CBM, PET microcomputer (Commodore Business Machines); PZT, piezoelectric transducer; DISK, discriminator.

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FLUCTUATIONS IN INTRACAVITY SECOND HARMONIC GENERATION

G.S. HOLLIDAY and Surendra SINGH
Department of Physics, University of Arkansas, Fayetteville, AR 72701, USA

Received 15 December 1986

Light intensity fluctuations in the fundamental mode when a second harmonic generating crystal is placed inside the laser resonator are discussed. An expression for the intensity probability density in the steady-state is derived and the effects of nonlinear interaction are discussed in terms of the moments of this distribution.

1. Introduction

In the process of second harmonic generation [1] a fundamental beam of frequency ω_1 interacts with its second harmonic of frequency $\omega_2 = 2\omega_1$ inside a nonlinear crystal. Under appropriate conditions pairs of photons from the fundamental beam are removed and converted into photons of the second harmonic. This process is widely used to extend the range of coherent radiation available from lasers. The process is also of interest in the generation of nonclassical states of the electromagnetic field [2]. Indeed the fundamental beam has been predicted to show antibunching after passing through a second harmonic generating crystal [3]. When the crystal is placed inside a resonator and pumped from an external source pronounced nonclassical effects are expected to occur. Such effects have been observed recently [4]. If the crystal is placed inside the laser resonator producing the fundamental beam the nonlinear interaction is expected to alter the fluctuation properties of the fundamental beam. In this paper we investigate the photon statistics of intracavity harmonic generation. We first derive the equation of motion for the density matrix of the fundamental. This equation is solved in the steady-state and intensity fluctuations are discussed in terms of the moments of this steady-state solution.

2. Equation of motion

Consider a nonlinear crystal placed inside the laser resonator of the fundamental beam. Because of nonlinear interaction some of the photons in the fundamental beam are converted to second harmonic photons. We assume that the second harmonic is also resonant, but, in a different cavity. The fundamental beam is also interacting with a set of two-level atoms that provide gain for the laser action to occur at the fundamental frequency. In addition, both the fundamental and the harmonic modes suffer losses due to the finite transmittivity of the resonator mirrors. The interaction hamiltonian for the system in the interaction picture will be

$$\hbar\hat{H} = \sum_{i=1}^{N} \hbar g(\hat{a}_1 \hat{b}_i^\dagger + \hat{a}_1^\dagger \hat{b}_i) + \tfrac{1}{2}\hbar\kappa(\hat{a}_1^2 \hat{a}_2^\dagger + \hat{a}_1^{\dagger 2} \hat{a}_2) \; . \tag{1}$$

The first term represents the resonant interaction of the fundamental beam with a set of N homogeneously broadened two-level atoms [5]. The coupling constant g is expressed in terms of the decay rates of the atomic levels, cavity volume and certain integrals of the mode-functions. The second term represents the interaction between the fundamental mode (subscript 1) and its second harmonic mode (subscript 2) inside the crystal.

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Inhomogeneously broadened laser with a saturable absorber

Mansour Mortazavi and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701

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A quantum theory for a laser with inhomogeneously broadened active and absorber atoms is presented. Photon-number distributions are derived for both on- and off-resonance operation of the laser, and the results are compared with the results of earlier investigations.

I. INTRODUCTION

It is well known that a nonlinear absorber inside a laser cavity imparts new characteristics to the laser which are of interest both theoretically and experimentally.[1–8] Examples of such lasers are the laser with a saturable absorber (LSA) and the dye laser. Both systems have been investigated in some detail. The treatments for the LSA with inhomogeneously broadened atoms are, however, valid either only for small intensities or do not fully take into account the effect of quantum noise. We would like to present a quantum-mechanical treatment that allows us to derive the photon-number distribution for an inhomogeneously broadened LSA. We also incorporate the effects of detuning and atomic motion.[9–11]

II. EQUATION OF MOTION

Consider a single-mode electromagnetic field at a frequency Ω inside a laser cavity interacting with a set of amplifying and another set of absorbing two-level atoms in gas phase. The distribution of atomic speeds along the resonator axis will be assumed to be Gaussian with root-mean-square speeds u_1 and u_2,

$$D_i(v)=\frac{1}{u_i\sqrt{\pi}}\exp\left[-\left[\frac{v}{u_i}\right]^2\right],\quad i=1,\,2\,. \tag{1}$$

Throughout this paper the subscripts 1 and 2 will refer to active and absorber atoms, respectively. The single atom-field Hamiltonian is given by[2]

$$\hat{H}=\hbar\Omega\hat{a}^{\dagger}\hat{a}+\sum_{i=1}^{2}[\hbar\omega_{0i}\hat{b}_i^{\dagger}\hat{b}_i+(\hbar g_i\hat{a}\hat{b}_i^{\dagger}+\text{H.c.})]\,. \tag{2}$$

Here $\hat{a}^{\dagger}(\hat{a})$ is the creation (annihilation) operator for the field, $\hat{b}_i^{\dagger}(\hat{b}_i)$ is the raising (lowering) operator for the atoms of type i, and ω_{0i} is the transition frequency. The coupling constant is given by $g_i=ex_i\sqrt{\Omega/2\epsilon_0\hbar V}$, where x_i is atomic transition dipole moment, Ω is the field frequency, and V is the quantization volume. The spatial variation of the field inside the cavity is taken into account by multiplying g_i by an appropriate (traveling or standing wave) mode function.

The equation of motion for the field density matrix is derived following the work of Scully, Kim, and Lamb[10] and Riska and Stenholm.[11] The details of this derivation can be found in the work of Roy.[2] Here we merely outline the various steps involved and point out differences that arise due to inhomogeneous broadening. We first calculate the change in the density matrix due to an atom introduced in state $|\alpha\rangle$. When this contribution is multiplied by r_α, the number of atoms introduced per second in $|\alpha\rangle$ and averaged over the distribution of detunings, we get the coarse-grained rate of change of the density matrix due to atoms introduced in state $|\alpha\rangle$. Similarly, passive losses are calculated by introducing a fictitious set of two-level atoms that absorb laser radiation. Adding the contribution from atoms and losses we obtain the following equation of motion for the probability $p(n)$ for finding n photons in the laser:[2]

$$\begin{aligned}\dot{p}(n)=&-[(n+1)R_1(n)+nR_2(n-1)+Cn]p(n)\\&+nR_1(n-1)p(n-1)\\&+[(n+1)R_2(n)+C(n+1)]p(n+1)\,,\end{aligned} \tag{3}$$

where

$$R_1(n)=\frac{2r_a|g_1|^2\gamma_{ab}}{\gamma_a}\int_{-\infty}^{\infty}dv\frac{D_1(v)}{(\omega_{01}-\Omega+kv)^2+\gamma_{ab}{}^2\left[1+\frac{4|g_1|^2}{\gamma_a\gamma_b}(n+1)\right]}\,, \tag{4}$$

$$R_2(n)=\frac{2r_d|g_2|^2\gamma_{cd}}{\gamma_d}\int_{-\infty}^{\infty}dv\frac{D_2(v)}{(\omega_{02}-\Omega+kv)^2+\gamma_{cd}{}^2\left[1+\frac{4|g_2|^2}{\gamma_c\gamma_d}(n+1)\right]}\,, \tag{5}$$

and C is the rate at which the field intensity decays. γ_α is the decay rate of level $|\alpha\rangle$ and $\gamma_{\alpha\beta}$ is the decay rate for the transition dipole moment between levels $|\alpha\rangle$ and $|\beta\rangle$. We have assumed that the active atoms are introduced in their upper state $|a\rangle$ and the absorber atoms are introduced in their lower state $|d\rangle$. The velocity integrals in Eqs. (4) and (5) appear because the atoms moving with different speeds will have different resonance frequencies due to the Doppler shift $kv=(\Omega/c)v$. The velocity integrals can be expressed in terms of plasma dispersion function.[12] Equations (3)–(5) are the basic equations of this paper.

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FIRST-PASSAGE-TIME STATISTICS FOR THE GROWTH OF LASER RADIATION

Marvin Young and Surendra Singh
Physics Department, University of Arkansas, Fayetteville, AR 72701

ABSTRACT

We report on the measurements of the first-passage-time distributions for the growth of laser radiation in a Q-switched laser near threshold.

The dynamics of a single-mode laser has been studied by a number of workers both theoretically[1] and experimentally.[2-4] Here we report on the measurements of yet another aspect of this problem, namely, the passage-time distributions for the laser intensity to grow from an initial value zero to a predetermined value for the first time. The time evolution of the complex field amplitude of the laser is governed by the nonlinear Langevin equation

$$\dot{E} = -\frac{\partial U}{\partial E^*} + \eta(t) \ , \tag{1}$$

where the potential $U(|E|^2)$ is given by

$$U(|E|^2) = -\tfrac{1}{2}a|E|^2 + \tfrac{1}{4}|E|^4 \ . \tag{2}$$

The Langevin noise term $\eta(t)$ represents spontaneous emission fluctuations and is taken to be a delta-correlated Gaussian random process with zero mean. The pump parameter a is proportional to the net gain. If the pump parameter is suddenly changed from a large negative value to some positive value (Q-switching), the light intensity grows from the initial value zero toward the nonzero steady-state intensity $I = |E|^2 = a$. The time taken by the light intensity to reach a predetermined threshold is called the first-passage-time (FPT). The FPT will fluctuate each time the laser is Q-switched reflecting the fact that the growth is initiated by the random spontaneous emission fluctuations.

Measurements of the FPT distributions were performed on a single-mode He:Ne laser operating at $\lambda = 633$ nm near threshold. The laser is turned on by applying a voltage pulse to an intracavity acousto-optic modulator. At the same time a gate allows pulses from a clock to reach a scaler. The growth of the light intensity is monitored by a photomultiplier tube (PMT) whose output is fed to a discriminator. When the PMT output voltage crosses the discriminator threshold (corresponding to some intensity) an output pulse is triggered which stops the gate. The number stored in the scaler is then a measure of the FPT. This number is transferred to a computer memory. By repeating this process a histogram of the FPT is built up. Figure 1 shows measured FPT distributions for several different pump parameters and for a fixed threshold $I_{th} = 2$. It is seen that with increasing pump parameter the peak of the distribution shifts to smaller values and its width decreases. These

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Laser theory without the rotating-wave approximation

Reeta Vyas and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701
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The effect of counter-rotating terms in the interaction Hamiltonian on the photon statistics is investigated for a single-mode laser. The treatment is based on the Scully-Lamb model. A perturbation expansion is used to derive an equation of motion for the field-density matrix. The counter-rotating terms are found to lead to both transient and secular effects. With some reasonable approximations the steady-state photon-number distribution is derived and it is shown that the counter-rotating terms tend to raise the threshold of laser action and broaden the photon-number distribution. The relative size of these effects is found to be very small and depends on the square of the ratio of the atomic linewidth and the transition frequency.

I. INTRODUCTION

In quantum optics, NMR, quantum electronics, and other branches of resonance phenomena it is customary to make the rotating-wave approximation (RWA).[1] Under this approximation certain terms which oscillate at twice the resonance frequency are dropped from the interaction Hamiltonian. The resulting Hamiltonian describes the dynamics of the system quite adequately. However, the rapidly oscillating terms (also called the counter-rotating or energy-nonconserving terms) can give rise to physical effects even if small. For example, in the Rabi problem they give rise to the Bloch-Siegert shift of the resonance frequency and small high-frequency amplitude modulation of the Rabi oscillations.[2] In laser theory the role of these counter-rotating terms has not been investigated. In this paper we wish to consider the effect of such terms in laser theory. We find that the counter-rotating terms give rise to both secular and time-dependent effects. In particular, their effect is to raise the threshold of laser oscillation and broaden the photon-number distribution. However, the magnitude of these corrections is very small as expected. In our investigations we adopt the model of Scully and Lamb[3] for a single-mode laser. Unlike them, however, we use a perturbative approach.[3,4] In Sec. II we describe the details of the model and derive a master equation for the density matrix of the laser field for a traveling-wave mode. In Sec. III, this master equation is solved in the steady state and changes produced by the presence of counter-rotating terms are discussed. We also consider the standing-wave case and discuss the differences that arise in the equations of motion.

II. EQUATIONS OF MOTION OF THE DENSITY MATRIX

We consider a single-mode electromagnetic field of frequency ω interacting with a group of N identical two-level atoms. The energy separation between the upper atomic level $|a\rangle$ and the lower atomic level $|b\rangle$ is $\hbar\omega_0$, where ω_0 is equal or close to the field mode frequency. The atoms may decay nonradiatively out of the two levels $|a\rangle$ and $|b\rangle$ to various other levels at a rate γ. Following Scully and Lamb[3] we suppose that the laser gain is provided by atoms introduced uniformly at an average rate $N_{\nearrow}$ throughout the cavity. These excited atoms interact with the field inside the cavity and make their contributions to the field one at a time. The total rate of change of the optical field may be calculated as a coarse-grained derivative by multiplying the change produced by one atom by the rate $N_{\nearrow}$ at which atoms are introduced in the excited state. Similarly, laser loss is simulated by considering another set of fictitious two-level atoms with broad levels introduced into the cavity in their lower state at a certain uniform rate.[3] These atoms absorb radiation and model the loss suffered by the laser field which is ultimately due to photons escaping from the cavity either because of the finite transmittivity of the mirrors or because of scattering from various elements inside the cavity. The Hamiltonian for an interacting atom and a single-mode electromagnetic field is given by

$$\hbar\hat{\mathscr{H}}=\hbar\omega_0\hat{R}_3+\hbar\omega\hat{a}^{\dagger}\hat{a}-2\boldsymbol{\mu}\cdot\hat{\mathbf{E}}(\mathbf{r},t)\hat{R}_1\ , \tag{1}$$

where $\hat{R}_1,\hat{R}_2,\hat{R}_3$ are the three Pauli spin-$\frac{1}{2}$ operators, $\boldsymbol{\mu}$ is the transition dipole moment between the levels $|a\rangle$ and $|b\rangle$ which we take to be real, and $\mathbf{r}$ is the position of the atom. The single-mode electric field operator $\hat{\mathbf{E}}(\mathbf{r},t)$ is given by (in mks units)

$$\hat{\mathbf{E}}(\mathbf{r},t)=\left[\frac{\hbar\omega}{2\epsilon_0 V}\right]^{1/2}[\boldsymbol{\epsilon}\hat{a}(t)u(\mathbf{r})+\boldsymbol{\epsilon}^*\hat{a}^{\dagger}(t)u^*(\mathbf{r})]\ , \tag{2}$$

where $\boldsymbol{\epsilon}$ is a unit polarization vector which we take to be real corresponding to linear polarization, $\hat{a}$ and $\hat{a}^{\dagger}$ are the annihilation and creation operators for the field, $u(\mathbf{r})$ is the cavity mode function, and V is the cavity volume (quantization volume). The third term in Eq. (1) represents the interaction Hamiltonian $\hbar\hat{H}$. This term in the interaction picture can be written as

$$\hbar\hat{H}=\hbar g\{[\hat{a}u(\mathbf{r})+\hat{a}^{\dagger}u^*(\mathbf{r})e^{2i\omega t}]\hat{b}^{\dagger}+\text{H.c.}\}\ , \tag{3}$$

where

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Decay of an unstable state

Marvin R. Young and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701
(Received 13 August 1984)

The first-passage-time problem associated with the decay of an unstable state is considered and analytic expressions for the first-passage-time distributions are derived. These expressions are quite general and are applicable to a large class of stochastic processes.

I. INTRODUCTION

The relaxation of a system from an initially unstable state towards a final stable state under the influence of random forces of microscopic origin is a fundamental problem in many branches of physics.[1-3] A more difficult but related problem is that of the relaxation of a metastable state.[4] These relaxation processes are examples of a stochastic process which eventually terminates when a certain variable characterizing the state of the system reaches a fixed threshold. The classic first-passage-time[5] (FPT) problem arises naturally in the description of the dynamics of such processes. Although this problem has a long and distinguished history and has been studied in many papers in various specific contexts,[6] the general problem has proved to be solvable for only the simplest of stochastic processes. In what follows, we wish to point out some general features of the FPT problem and show that for a large class of stochastic processes the FPT probability density can be represented by one or the other of the two analytic expressions given below. In the following we first present the results for one-dimensional problems and then generalize them to multidimensional processes with isotropic diffusion.

II. FPT PROBLEM IN ONE DIMENSION

Consider a random process $x(t)$ in one dimension whose probability density $p(x,t)$ obeys a Fokker-Planck equation

$$\frac{\partial p(x,t)}{\partial t}=-\frac{\partial}{\partial x}A(x)p(x,t)+\frac{1}{2}\frac{\partial^2}{\partial x^2}2Dp(x,t)\ , \quad (1)$$

where $2D$ is the diffusion coefficient which is also the noise intensity and $A(x)$ is the drift coefficient. The detailed dependence of $A(x)$ on x is not of interest to us. Some restrictions on $A(x)$ will be specified later. The steady-state solution of Eq. (1) corresponding to a vanishing probability current density at infinity is given by

$$p_s(x)=Q^{-1}\exp\left[-\frac{U(x)}{D}\right],\quad U(x)=-\int^x A(x)dx\ , \quad (2)$$

where Q is a normalization constant. Now consider this problem in some interval $\Omega\{x: x_1\le x\le x_2\}$. We are interested in the time T that it takes the process $x(t)$ to reach the boundary of Ω for the first time if it started at some point x_0 inside Ω. The variable T itself will be a random process described by a probability density $P(T;x,x_0)\equiv P(T)$ so that $P(T)dT$ is the probability that a first passage of $x(t)$ through x occurs between T and $T+dT$ if initially $x(t)=x_0$. The FPT probability density $P(T)$ is given by[5]

$$P(T)=-\frac{\partial}{\partial T}\int_{x_0}^{x}G(x,t_0+T;x_0,t_0)dx\ , \quad (3)$$

where $G(x,t_0+T;x_0,t_0)\equiv G(x,T;x_0,0)$ is the Green-function solution to Eq. (1) with boundary conditions

$$G(x,t_0;x_0,t_0)=\delta(x-x_0)\ ,$$
$$G(x,t_0+T;x_0,t_0)\,|_{x=x_{1,2}}=0\ . \quad (4)$$

This Green function has the following eigenfunction expansion[1,5]

$$G(x,t;x_0,t_0)=\left[\frac{p_s(x)}{p_s(x_0)}\right]^{1/2}\sum_n \phi_n(x)\phi_n(x_0)e^{-\lambda_n(t-t_0)D}\ ,\quad t\ge t_0 \quad (5)$$

where $\phi_n(x)$ and λ_n are the eigenfunctions and the eigenvalues of a one-dimensional Schrödinger equation associated with Eq. (1),

$$\left[\frac{d^2}{dx^2}+\lambda_n-V(x)\right]\phi_n(x)=0\ , \quad (6a)$$

where

$$V(x)=-\frac{1}{2}\frac{U''(x)}{D}+\frac{1}{4}\left[\frac{U'(x)}{D}\right]^2 \quad (6b)$$

and $\phi_n(x)$ satisfy the boundary conditions

$$\phi_n(x_{1,2})=0\ . \quad (6c)$$

Here the primes denote differentiation with respect to x and the boundary conditions (6c) are a consequence of Eqs. (4) and (5). Equation (6) defines a Sturm-Liouville problem and the eigenfunctions $\phi_n(x)$ form an orthonormal complete basis in Ω.

The unstable and the stable states of the system corre-

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STATISTICAL PROPERTIES OF SINGLE-MODE AND TWO-MODE RING LASERS

Surendra SINGH

Department of Physics, University of Arkansas, Fayetteville, AR 72701, U.S.A.

Received December 1983

Contents:

Abstract:

Recent theoretical and experimental investigations of the statistical properties of single-mode and two-mode ring lasers are described. A theory of the frequency dependence of laser light fluctuations in a single-mode laser and the photoelectric experiments testing the predictions of this theory are discussed. For the case of two-mode ring lasers, a theoretical discussion of the steady-state and transient properties, such as photon statistics, first-passage-time distributions, correlations and phase locking, is given. Theoretical predictions are compared with the results of photoelectric measurements carried out on two-mode gas and dye ring lasers. A summary and the principal conclusions of this article are presented, and suggestions for further investigations are made.

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CHAOS IN COHERENT TWO-PHOTON PROCESSES IN A RING CAVITY

Surendra SINGH
Department of Physics, University of Arkansas, Fayetteville, AR 72701, USA

and

G.S. AGARWAL
School of Physics, University of Hyderabad, Hyderabad 500 134, India

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The output of a ring cavity containing a resonant medium undergoing two photon transitions is shown to become chaotic, after following a series of bifurcations involving 2^n cycles, as the strength of the driving field is increased. The chaotic regime is followed by a sequence of period doubling bifurcations in reverse order.

The resonant interaction of the atoms, contained in a cavity, with the self-consistent fields in the cavity leads to a variety of instabilities [1–6]. For example, Ikeda et al. [1] predicted the chaotic behavior of the output of a ring cavity for certain values of the input field strength and the parameter of the system. The type of the chaotic character predicted by Ikeda et al. has been observed by Gibbs et al. [7] in experiments involving nonlinear feedback mechanisms such as those involving nonlinear electrooptic modulators. The transition to chaos has been shown [1,4] to follow the Feigenbaum scenario [8,9]. This interrelation of the "optical turbulence" and the chaotic character exhibited by various mathematical models [8–10] is quite exciting and one is led to think that other types of nonlinear interactions of atoms with the fields in the cavity might exhibit similar turbulent or chaotic behavior. In order to explore this possibility, we have analyzed the problem of coherent two photon transitions [6,11–13] in a ring cavity. We show that the path leading to chaos in such transitions again follows Feigenbaum's scenario in spite of the very complicated two dimensional map that characterizes the coherent two photon transitions in a ring cavity.

Our treatment is based on the coupled Maxwell–Bloch [13] equations for coherent two photon processes with appropriate boundary conditions. We use a method similar to that of Ikeda et al. in order to obtain the two dimensional dynamical map that characterizes the output of the ring cavity produced in coherent two photon transitions.

The Maxwell–Bloch equations for two photon absorption processes [11–13] are given by

$$\partial S/\partial\tau = -\gamma_\perp S + \mathrm{i}(\omega_{12} - 2\omega)S + \mathrm{i}g\mathcal{E}^{*2}W, \tag{1a}$$

$$\partial W/\partial\tau = -\gamma_\parallel(W+1) + \tfrac{1}{2}\mathrm{i}(g^*\mathcal{E}^2 S - \text{c.c.}), \tag{1b}$$

$$\partial\mathcal{E}/\partial z = (\pi\omega N v\hbar/c^2)\,\mathrm{i}gS^*\mathcal{E}^*. \tag{1c}$$

In writing (1) we have ignored the Stark shifts. We have also introduced the reduced time $\tau = t - z/v$. We assume that $\gamma_\perp \gg \gamma_\parallel$, so that the variable S relaxes quickly to the equilibrium value

$$S = \mathrm{i}g\mathcal{E}^{*2}W/\gamma_\perp(1-\mathrm{i}\delta), \quad \delta = (\omega_{12} - 2\omega)/\gamma_\perp. \tag{2}$$

We then obtain

$$\frac{\partial\mathcal{E}}{\partial z} = \frac{\pi\omega N\hbar}{c}\frac{|g|^2}{\gamma_\perp(1+\mathrm{i}\delta)}|\mathcal{E}|^2\mathcal{E}W, \quad v \approx c, \tag{3a}$$

$$\frac{\partial W}{\partial\tau} = -\gamma_\parallel(W+1) - \gamma_\parallel\left(\frac{|g|^2}{\gamma_\perp\gamma_\parallel}\right)\frac{|\mathcal{E}|^4 W}{1+\delta^2}. \tag{3b}$$

In terms of an auxiliary quantity ψ defined by

ANTIBUNCHING, SUB-POISSONIAN PHOTON STATISTICS AND FINITE BANDWIDTH EFFECTS IN RESONANCE FLUORESCENCE

Surendra SINGH
Department of Physics, University of Arkansas, Fayetteville, AR 72701, USA

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It is demonstrated that, although the fluorescent photons from a driven two-level atom always exhibit antibunching, they need not obey sub-poissonian statistics. The maximum sub-poissonian excursion of the normalized second factorial moment $(\langle(\Delta n)^2\rangle - \langle n\rangle)/\langle n\rangle$ occurs in the transient regime where it can approach the value -1. Effects of finite bandwidth of excitation on photon statistics are also investigated and it is shown that the photon number distribution may be narrower with finite bandwidth excitation than with coherent excitation.

The phenomenon of antibunching [1,2] exhibited by photons emitted in resonance fluorescence is an explicit feature of a quantized electromagnetic field. Another non-classical feature of the electromagnetic field in resonance fluorescence, the so-called squeezing, was discussed by Walls and Zoller [3]. The relation between photon statistics and these non-classical features has been investigated recently. It was shown by Mandel [4] that in the steady-state, both antibunching and squeezing lead to sub-poissonian photon statistics. We will show that although the fluorescent photons from a two-level atom always exhibit antibunching, they may not always obey sub-poissonian statistics. Indeed, for large counting times they may obey super-poissonian statistics. We also take into account the finite bandwidth of the exciting field and show that a finite bandwidth excitation may lead to a narrower photon number distribution than a coherent excitation.

The probability distribution $p(n, 5, t+T)$ for the number of photons n emitted in a given time interval $(t, t+T)$ by a driven two-level atom has been investigated by Mandel [5], Cook [6], and Lenstra [7] for coherent excitation under some special circumstances. In the following we will assume the exciting field to be a constant amplitude field whose phase performs a random walk. This phase diffusion model [8] is characteristic of a laser operating far above threshold and leads to a lorentzian spectrum with a certain bandwidth λ (HWHM). The result of coherent excitation can then be obtained by taking the limit $\lambda \to 0$. To calculate $p(n,t,T)$ we note that the photoelectric counting formula reproduces the photon number distribution $p(n,t,T)$ if the quantum efficiency of the detector is unity. Accordingly, we may write [9]

$$p(n,t,T) = \left\langle\!\!\left\langle \mathcal{T} : \frac{1}{n!}\left[\int_t^{t+T} \hat{I}(t')\,\mathrm{d}t'\right]^n \exp\left[-\int_t^{t+T} \hat{I}(t')\,\mathrm{d}t'\right] : \right\rangle\!\!\right\rangle,$$

where :: and $\mathcal{T}$ are normal ordering and time ordering symbols and $\hat{I}(t)$ is total photon flux in units of photons per second. The brackets in eq. (1) indicate the expectation value with respect to the state of the system and average over all realizations of the phase of the exciting field. From eq. (1) we find that the rth factorial moment of n is given by

$$\langle n^{(r)}\rangle = r! \int_t^{t+T} \mathrm{d}t_r \int_t^{t_r} \mathrm{d}t_{r-1} \cdots \int_t^{t_2} \mathrm{d}t_1 \langle\!\langle : \hat{I}(t_r)\hat{I}(t_{r-1}) \cdots \hat{I}(t_2)\hat{I}(t_1) : \rangle\!\rangle\,,$$

$$t_r > t_{r-1} > t_{r-2} \cdots t_1; \quad r = 2, 3 \ldots .$$